Matrices

Friday, 4 January 2019 11:18 am

By the end of the lesson I would hope that you have an understanding of the concepts below which you can apply to a number of complex questions:

- Know how to express something in matrix notation
- Add, subtract and multiple matrices by a scalar
- Multiply matrices together
- Use the CAS to do all of the above

RECAP

I've mentioned matrices a lot in previous videos for this course.
They are brilliant and can be used to help us find solutions to simultaneous equations. But, we're not quite there yet!
Whilst Matrices has not been seen before, they are awesomely easy and a great tool which is used later in the course when we get to:

- Solving simultaneous equations
- Transformations

The language of Matrices

There really isn't much to really know.
As usual, in mathematics, there are a lot of terms and words used to describe things.

Matrix (array of numbers)

$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ $\begin{bmatrix} 2 & 1 \end{bmatrix}$ $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 1 & 0 & 1 \end{bmatrix}$ $\begin{bmatrix} 0 \end{bmatrix}$

Naming matrices

$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$ $A + B$

Entries (in Further I believe they are called elements!):

$X = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

	A	B	C	D
1				
2				
3				
4				

Dimension (size)

$Z = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ 2 by 2 (2×2) R C $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ 2×1

Rows and columns

Elements can be defined using algebraic notation: a_{ij} can be used

$a_{ij} \rightarrow A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$ $\quad a_{11} = 1$, $a_{21} = 4$

$a_{ij} = i + j \quad \begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}$

Adding, subtracting and multiplying by a scalar

To be able to add and subtract matrices, they have to be of the same **order** or have the same **dimension.**

$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $\quad A + B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 5 & 6 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 6 & 8 \\ 5 & 4 \end{bmatrix}$ $\quad$ 2 by 2

$B = \begin{bmatrix} 5 & 6 \\ 2 & 0 \end{bmatrix}$ $\quad B - A = \begin{bmatrix} 5 & 6 \\ 2 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ -1 & -4 \end{bmatrix}$

$3A \quad 3\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 6 \\ 9 & 12 \end{bmatrix}$

$3A + 2B$

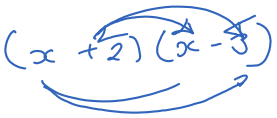
To multiply by a scalar you are simply multiplying each element or entry by the number outside of the matrix.

Multiplication of Matrices

Let's start easy!

Let's have a look at how it's done on the CAS.
Then look at how it's done by hand.

Firstly, remember FOIL?



11
12
21
22

The same process holds true for multiplying Matrices.

Firstly, some **rules**:

1. You can only multiply matrices when the number of columns in the first matrix matches the number of rows in the second matrix.
2. The size of the resulting matrix is the number of rows in the first matrix by the number of columns in the second matrix.
3. You multiply rows by columns.
4. You multiply individual entries/elements per row/column and add the results.

11 ✓
12 ✓
21 ✓
22

Sounds complicated doesn't it?!

Let's make number 1 and 2 easier …

$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $\quad A.B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 8 & 4 \\ 18 & 8 \end{bmatrix}$

$B = \begin{bmatrix} 2 & 0 \\ 3 & 2 \end{bmatrix}$ $\quad (2\times 2)\ (2\times 2)$

$C = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ $\quad A.C = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 7 \\ 15 \end{bmatrix}$

$(2\times 2)\ (2\times 1)$ ✓ ✓

Now here are some examples of how to multiply two matrices together:

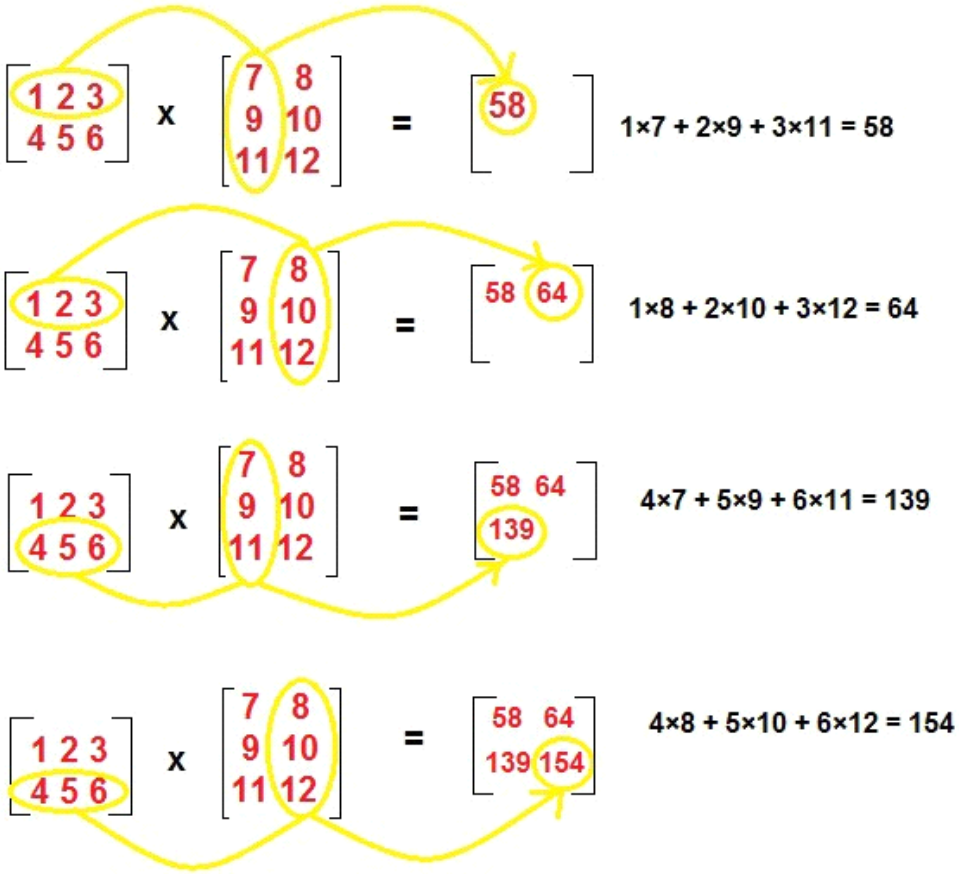
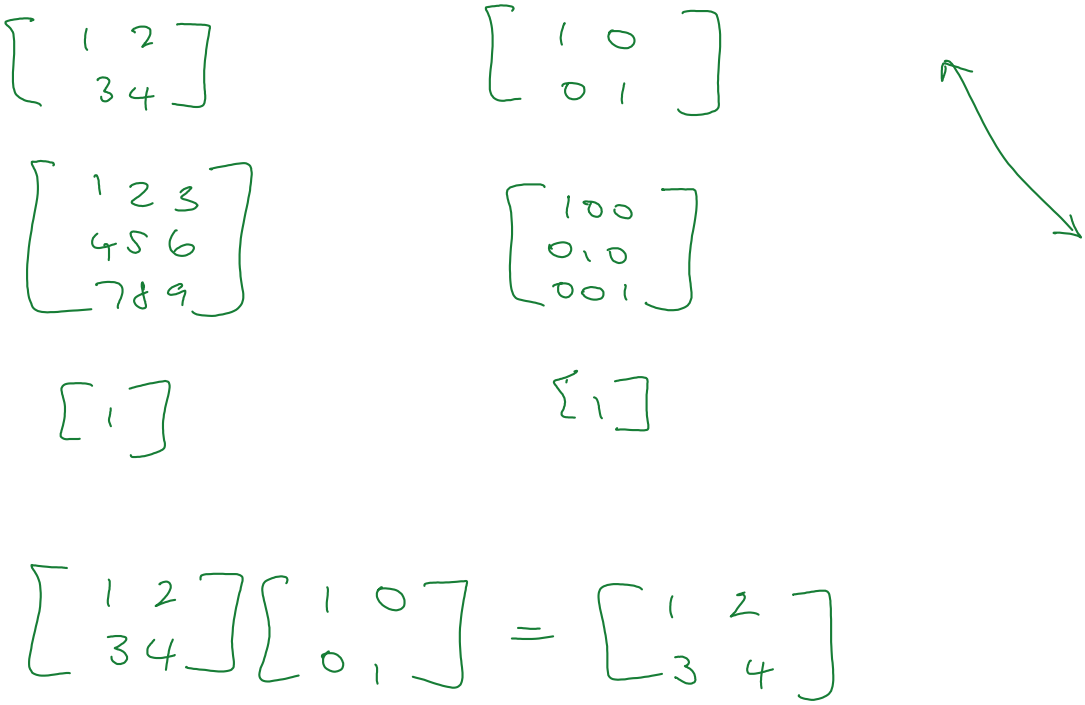
$C.A = \begin{bmatrix} 1 \\ 3 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$(2\times 1)\ (2\times 2)$

A visual example to help

This was taken from the inter-web somewhere and I think it's awesome!

It shows how to multiple two matrices together.



Finally ... Identity Matrix and Square Matrix