Logarithms

Sunday, 27 May 2018 5:48 pm

By the end of the lesson I would hope that you have understood and can apply the following learnings:

- Why we need logarithms
- What it means to be expressed in terms of logarithms
- How to express powers as logarithms
- How to use the common rules for logarithms to simplify expressions and equations

RECAP:

Generally speaking we can sketch sensible looking graphs using a uniform scale and show a relationship.
We have been dealing with lots of these graphs.
However, what if something doesn't go up in nice divisions.
What if the initial values are really small, and then grow larger and larger by factors of 10, 100, 1000 etc?
This would mean that we might have a graph which, if we were to try and draw a uniform scale, wouldn't fit.

Hence, we needed a way to represent really large numbers on a scale where we could find patterns.

x	1	10	100	1000	10000	100000
$\boxed{\text{power of } 10}$	10^0	10^1	10^2	10^3	10^4	10^5
x	0	1	2	3	4	5

$\log_{10}$ $\log_3$

Turning large numbers into smaller numbers is actually quite useful!

Logarithms and Bases

First thing to note is that logs can have bases.
These bases are pretty important

Example:
$\log_3 9 = 2$ means, "*What power of three gives an answer of 9?*" → $\boxed{\log_3} 9 = 2$
The base is the number you have a power of.

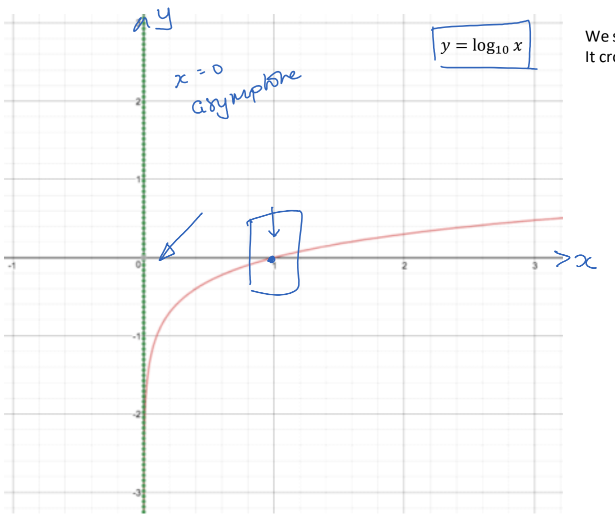
Example:
$\log_{10} 1000 = 3$ — $\boxed{\log_{10}} 1000 \rightarrow 3$

$\boxed{\log_9} 1 = 0$

Important:
$\log_a 1 = 0$ $\underline{\log_a} 1 = 0$

What does a logarithm graph look like?

If we use DESMOS to sketch it we see the following:



We see it has an asymptote at $x = 0$
It crosses the x-axis at the point (1, 0)

$\log_{10} 1 = 0$

Using some algebra we know that we can write: $y = \log_{10} x$ in a different way by using the thinking above:

$\boxed{10^y = x}$

Which looks pretty familiar if we compare it to:

$y = \boxed{\log_{10}} x$

$y = 10^x$

Which looks pretty familiar if we compare it to:

$y = 10^x$

$y = \log_{10} x$

$10^y = x$

In fact, it would appear to be the inverse of each other!!!!

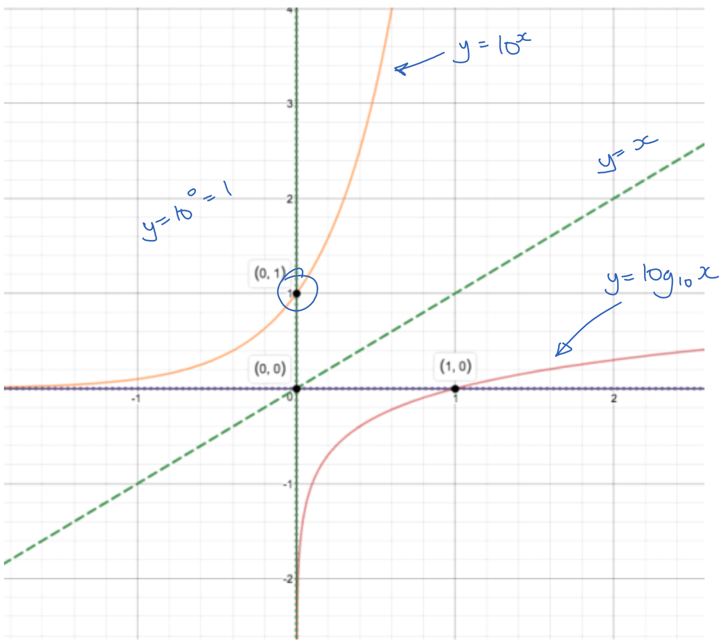
And, so it can be shown that $y = 10^x$ and $y = \log_{10} x$ are in fact the inverse functions of each other:

$\boxed{\log_9} 81 = \boxed{2}$

$9^2 = 81.$

$10^y = x$

$10^x = y$



It is really important to note that for a logarithmic function it is not defined for any values less and or including 0.

The special (and much used) graph of $y = e^x$

Remember, from the last lesson, this graph of $y = e^x$ is pretty important to us.
In the same way that we can have an inverse for $\log_{10} x$ we can also have an inverse for $y = e^x$

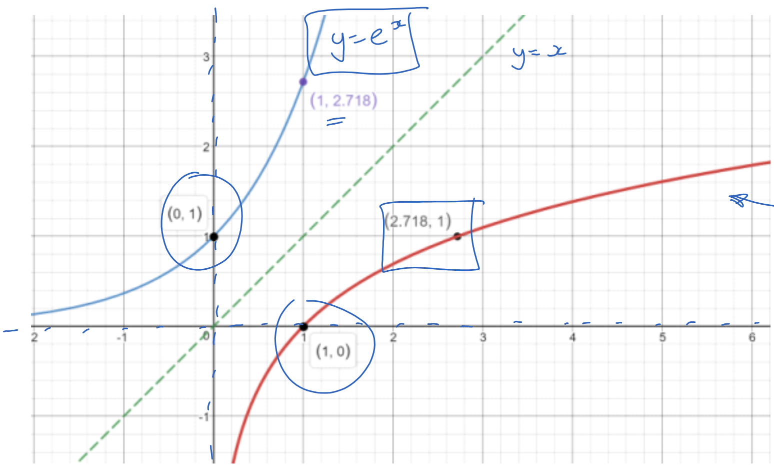
The Natural log:

The inverse of $y = e^x$ is $y = \log_e x$ which makes sense really!

Being Mathematicians, we like to write things as simply as we can.

Hence, $\log_e x$ can also be expressed as $\ln x$. **They mean exactly the same thing!**

$y = e^0$



$y = \log_e x$

$\log_e = \ln$

$y = \ln x$

Decimal values and their logs

$\log_{10} 10 = 1$

$\log_{10} 0.1 = -1$ because 10 to the power of -1 is one tenth.

$\boxed{\log_{10}} 10 = 1$ $\quad$ $\boxed{\log_{10}} 0.1 = -1$ $\quad$ $10^? = 10^{-1}$

RECAP: Index laws

Remember from a previous lesson we had the index laws given to us as:

- $a^n \times a^m = a^{n+m}$
- $a^n \div a^m = a^{n-m}$
- $(a^x)^y = a^{xy}$
- $(ab)^x = a^x b^x$
- $\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$
- $a^{-x} = \frac{1}{a^x}$
- $a^0 = 1$

$x^2 \times x^3 = x^5$

$x^{10} \div x^7 = x^3$

$x^0 = 1$

$(ab^2)^3 = a^3 b^6$

There just so happen to be a number of laws for logarithms! Such fun!!

Logarithm laws

Law 1: $\log_a(mn) = \log_a m + \log_a n$

Law 2: $\log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n$

Law 3: $\log_a(m^p) = p\log_a m$

Law 4: $\log_a(m^{-1}) = -\log_a m$

Law 5: $\log_a 1 = 0$ and $\log_a a = 1$ *

$\log_a m + \log_a n = \log_a(mn)$

$\log_{10} 3 + \log_{10} 6 = \log_{10}(3\times 6) = \log_{10} 18$

$\log_a m - \log_a n = \log_a\left(\frac{m}{n}\right)$

$\log_a m^{p} = p \times \log_a m$

$2\log_{10} 3 = \log_{10} 3^{2} = \log_{10} 9$

$\log_9\left(\frac{1}{9}\right)$

$\log_9(9^{-1})$

$= -1\times\boxed{\log_9 9}$

$= -1\times 1$

$= -1$

Funky logarithm equations

Let's get funky with logs and equations … not like there won't be lots of these on the exams or anything!

Example:
Find x if $\boxed{\log_2} x = 6$

$2^6 = x$

$x = 64$

$\cancel{\log_2} x = 6$

$x = 2^6$

$x = 64$

Example:
Find x if $\boxed{\log_3}(2x+1) = 3$

$3^3 = 2x+1$

$2x+1 = 27$

$2x = 26$

$x = 13$

$\cancel{\log_3}(2x+1) = 3$

$2x+1 = 3^3$

$2x+1 = 27$

$x = 13$

Example:
Solve the following to find the value of x:
$\log_e(x-1) + \log_e(x+2) = \log_e(6x-8)$

$\cancel{\log_e}(x-1)(x+2) = \cancel{\log_e}(6x-8)$

$x^2 + 2x - x - 2 = 6x - 8$

$x^2 + 2x - x - 2 - 6x + 8 = 0$

$x^2 - 5x + 6 = 0$

$x^2 - 2x - 3x + 6 = 0$

6	
1	6
-2	-3

$x(x-2) - 3(x-2) = 0$

$(x-3)(x-2) = 0$

$x - 3 = 0$ or $x - 2 = 0$

$x = 3$ $\quad$ $x = 2$

Note:
When solving functions you must ALWAYS check the reasonableness of the answers.
You cannot have negative values of x!

Example:
Solve the following to find the value of x:

$\log_2 x - \log_2(7-2x) = \log_2 6$

$\cancel{\log_2}\left(\frac{x}{7-2x}\right) = \cancel{\log_2} 6$

$\frac{x}{7-2x} = 6$

$x = 6(7-2x)$

$x = 42 - 12x$

$13x = 42$

$x = \frac{42}{13}$