Identifying and describing relations and functions

Tuesday, 20 November 2018 11:53 AM

By the end of the lesson I would hope that you have an understanding of the concepts below which you can apply to a number of complex questions:

- What is meant by the following terms:
  - Relation
  - Domain
  - Range
  - Image
  - Pre-image
- Know what a function is and how to read function notation
- Know how to test if a relation is a function using the Vertical Line Test
- Know what it means to restrict a function

RECAP:

As this is the start of a new course, there really isn't much to be able to recap.
However, there is a lot of language in Mathematics which we need to be completely conversant with.
This lesson looks at some of the most important language which will be used throughout the course.

Relationships

It's funny the lengths we will go to (as human beings) to try and find love ... or fame!
Clip of the Bachelorette

Thankfully Maths has this all sorted and you've been a part of many happy relationships.
Below is an example:

$$y = 3x + 4$$

This couple are blissfully happy!

$x = 3$

$y = 13$

OK.
There isn't much in terms of love ...
But there is a relationship between the values of x and y.

If I provide the equation above an x value then I will gain a y value.
There is a link.

In mathematics a **relation** is a **set of ordered pairs**.

What is an ordered pair?

Easy!
You've met thousands of them.
We called them something else.

$$(3, 8)$$

x y

This is an ordered pair.
Otherwise known as a coordinate

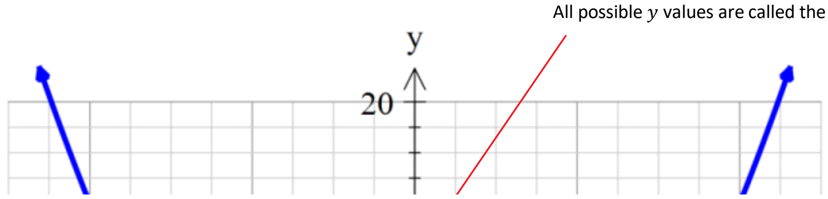
Note: x is known as the **pre-image**
Note: y is also known as the **image**.

Ordered pairs, gained from an equation, would form a set.

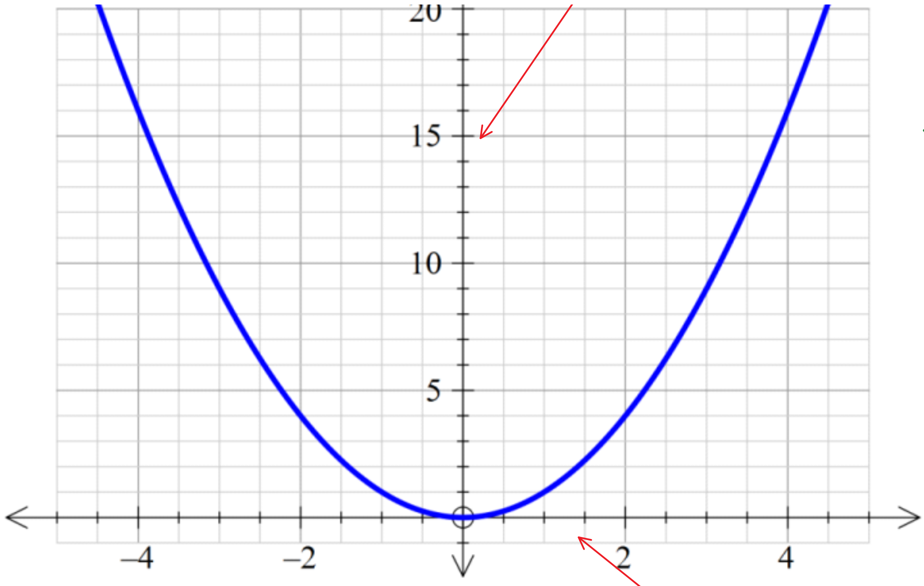
Domains

This is something I know lots of students find confusing.
I have to admit to not knowing why :(

All possible y values are called the range



$y = x^2$



$y = x^2$

All possible x values are the **domain** of the function

In the above function we can see that all x values are possible for the graph of $y = x^2$. Hence the **domain** would be described as:

$$x \in \mathbb{R}$$

$x \in \mathbb{R}$

This can also be expressed as:

$$x \in (-\infty, \infty)$$

$(-\infty, \infty)$ $\boxed{x \in \mathbb{R}}$

The important thing to note about the **domain** is that you include the values which are possible to be placed into the function.

The **range** is exactly the same idea. However, the range includes all possible values which could be created from the domain.

If we look at the function above ($y = x^2$) we can see that whilst all x values are possible, these only create values ranging from Zero to Infinity.
This can be expressed as:

$$f(x) \in \mathbb{R}^+ \cup \{0\}$$

$f(x) = y$

Or, alternatively as:

$$f(x) \in [0, \infty)$$

$f(x) \in [0, \infty)$

Note the use of the **square bracket**
This means that **zero** is included.

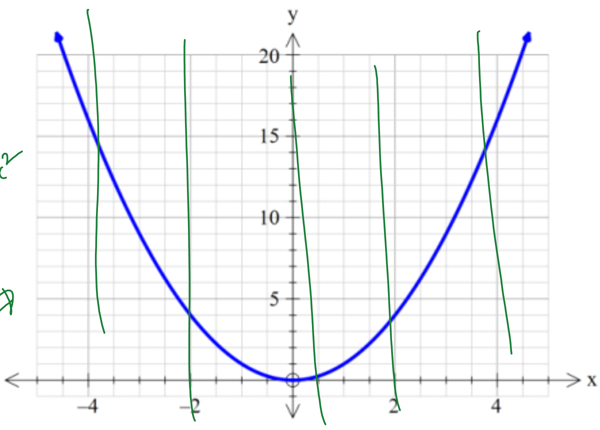
Functions, functions and more functions

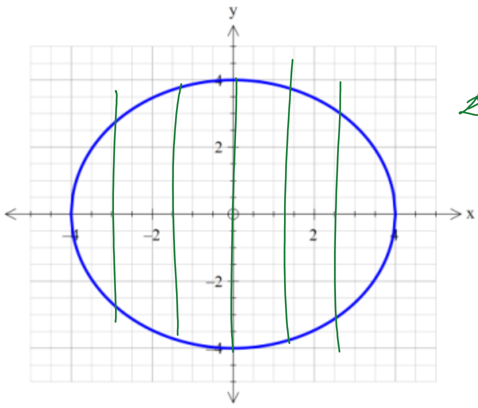
Maths is a BIG FAT TRICK.
There are many questions in exams which will test your understanding of what a function is versus what a relation is.

- **Relation**: All equations express a relationship.
- **Functions**: These are a very special kind of relationship

A function is defined such that for each x-value there is one one corresponding y-value.

$y = x^2$





This is an example of a function.
Each x-value has only one y-value.

This is not a function.
It is a relation.

Is there an easy way to decide if something is a **function**?
Yes!
It's called the **vertical line test**.

If we draw a vertical line and it only cuts the relation once, then it is defined as a function.

Easy!

Function Notation

This is so important.
When doing SACs and exams, you will be required to write functions using function notation.
This has a specific form.
An example is shown below:

$$f: [-4,4] \to \mathbb{R}, f(x) = x^2$$

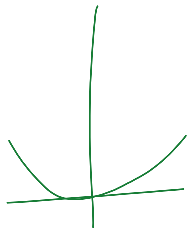
$[-4,4] \to \mathbb{R}$

Function is defined using a lower case letter. This matches what is at the end

Domain: Values of x for which the function is to be drawn

Co-domain

The function



Another example:

$$g: \mathbb{R} \to \mathbb{R}, g(x) = 3\sin(2x + \frac{\pi}{4}) - 2$$

$y =$

Being a trick we can change the way we express functions!

$$\{\ (x, y) :\ y = x^2, x \in [-4,4]\ \}$$

$$f: [-4,4] \to \mathbb{R}, f(x) = x^2$$

Restricting a function

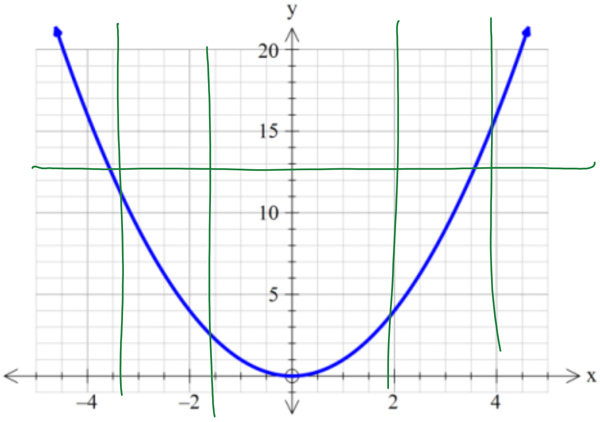
Later on, in this chapter, and others we are going to meet something called a **one-to-one function**.
This is a subset of a function.

Remember: A function is a relation which passes the vertical line test.

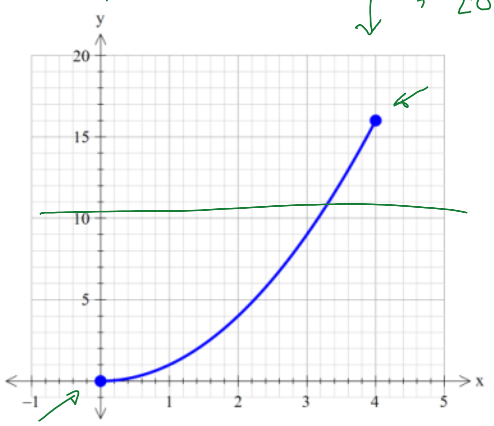
A one-to-one function must also pass a **horizontal line test.**

If we can draw a **horizontal line** through the function and it only cuts once, then the function is a one-to-one function

$f: [0,4] \to \mathbb{R},\ f(x) = x^2$



This is **NOT** a one-to-one function.



This IS a **one-to-one** function

As it turns out … they are both the graph of $y = x^2$.
The second graph has just had a **restricted domain** where we have effectively cut the graph to ensure that, when we cut it with a horizontal line, it only cuts the function once.

Hence, the restricted domain for the function would look like:

$$f: [0, 4] \to \mathbb{R}, f(x) = x^2$$