Graphing Logarithmic Functions

Thursday, 31 May 2018 10:57 AM

By the end of the teaching of this module I would hope that the following has been covered and understood. Hopefully you will be able to apply the learning to any exam questions:

- Know how to draw the base graphs for logarithms
- Know how to transform the base graphs to get to more complex graphs
- Know how to change the base of logarithmic functions
- Know how to transform one exponential graph to another

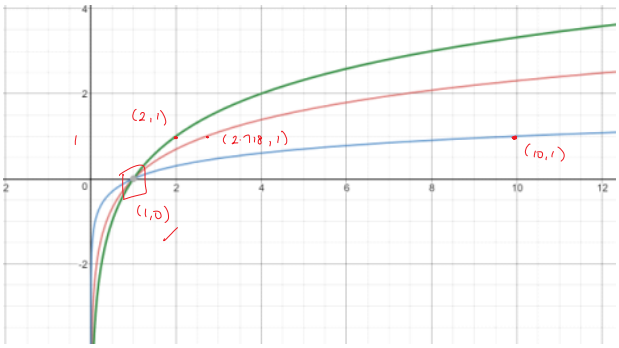
RECAP:

We have been looking exponentials and logs.
We have looked how what graphs of exponentials look like and how we can solve exponential functions.
We then moved on to look at solving exponential functions as well has how to change exponential notation into power notation and vice versa.

This lesson takes a look at graphing graphs of logarithmic functions.

Graphing Logarithmic Functions

The following are examples of log functions:

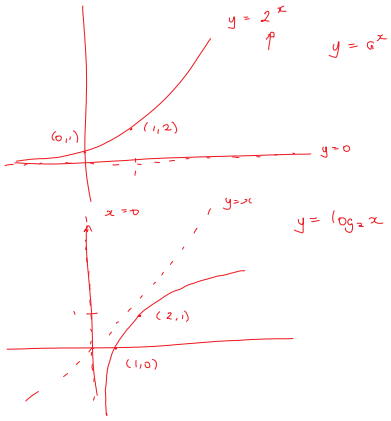


Blue: $y = \log_{10} x$

Red: $y = \log_e x$

Green: $y = \log_2 x$

Maximal Domain: $\mathbb{R}^+$
Range: $\mathbb{R}$
Y-axis is a vertical asymptote



$y = \log_2 x$

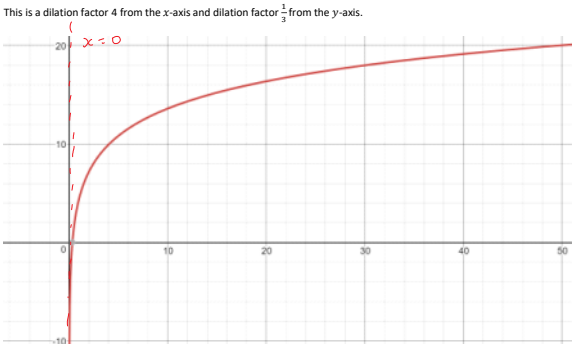
Transforming log functions

We have done a lot in terms of transforming functions.
This theory is now applicable to the log functions.

Logs can be dilated, reflected and translated in exactly the same way as all other functions.

Example: $y = 4\log_e(3x)$

This is a dilation factor 4 from the x-axis and dilation factor $\frac{1}{3}$ from the y-axis.



Looking at important points:

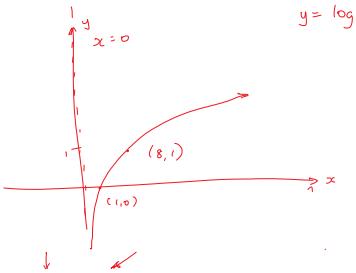
$(1, 0) \to \left(\frac{1}{3}, 0\right)$

$(e, 1) \to \left(\frac{e}{3}, 4\right)$

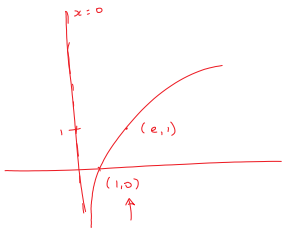
Domain: $\mathbb{R}^+$ $(0, \infty)$

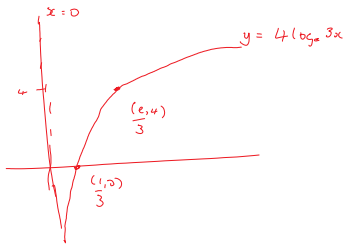
Range: $\mathbb{R}$

$y = \log_8 x$

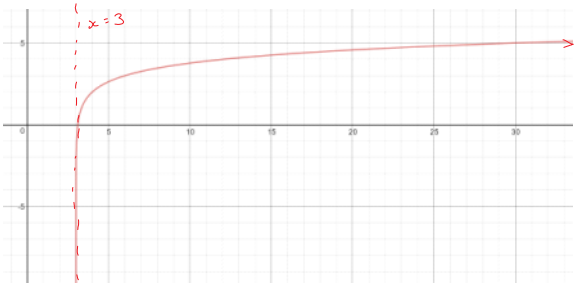


$y = 4\log_e 3x$





Example: $\log_3(x - 3) + 2$



Important points:

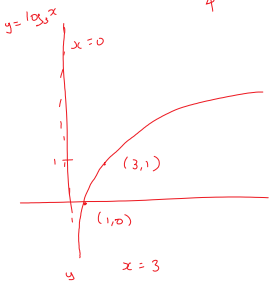
$(1, 0) \to (4, 2)$

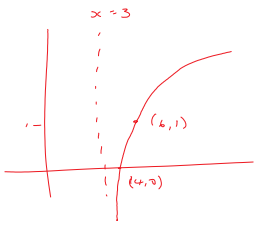
$(3, 1) \to (6, 3)$

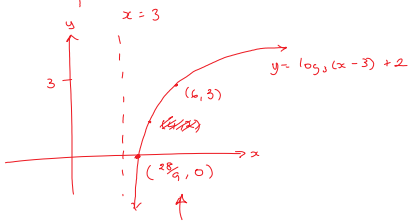
Domain: $(3, \infty)$

Range: $\mathbb{R}$

$y = \log_3(x - 3) + 2$







Important to note:

When drawing graphs, it's important to always make sure you do the following:

- Label axes
- Label (as coordinates) axis intercepts
- Label at least one coordinate outside of the above
- Label turning points
- Label the graph with the equation (if more than one equation exists on a set of axes)
- Label endpoints (with the correct open or closed circle)

As this is a CAS course, we can use the CAS to help us find axis intercepts.

Remember:

- To find x-axis intercepts, make $y = 0$and solve
- To find y-axis intercepts, make $x = 0$ and solve

It is always important to ensure you write the coordinates in the form the questions is expecting.
In Methods, this is always exact value unless otherwise stated.

Example: $(0, 2\log_e 5 - 3)$ is a perfectly acceptable exact form of a coordinate.

$(0, 2\log_e 5 - 3)$

RECAP: Changing bases from Methods 1 and 2

In the Methods 1 and 2 course we have met the following idea in that we can one function into two bases:

$\log_{10} x$

$\log_3 4 = \frac{\log_{10} 4}{\log_{10} 3}$

RECAP: Changing bases from Methods 1 and 2

In the Methods 1 and 2 course we have met the following idea in that we can one function into two bases:

$$\log_a x = \frac{\log_{10} x}{\log_{10} a}$$

Note that the lower number is on the bottom of the fraction!

$\log_{13}(4) = \frac{\log_{10} 4}{\log_{10} 13}$

$y = \frac{x}{2} \Rightarrow y = \left(\frac{1}{2}\right)x$

This is the same as writing it as ...

$$\log_a x = \frac{1}{\log_{10} a} \times \log_{10} x$$

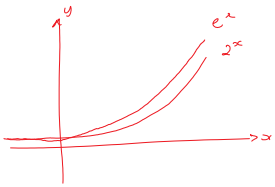
$y = \frac{x}{a}$

$y = \left(\frac{1}{a}\right) \times x$

This is nothing more than a dilation of Factor $1/(\log_{10} a)$ away from the x –axis

Sometimes we might be asked to transform from one exponential function to another! For example:

Find a transformation which takes the function $y = 2^x$ to $y = e^x$



$2^x = 6$

$\log_2 2^x = \log_2 6$

$x = \log_2 6$

$x = \log_2 6$

$2^x = 2^{\log_2 6}$

$y = 2^x \longrightarrow y = e^x$

$e = 2^{\log_2 e}$

$(a^b)^c = a^{bc}$

$e^x = \left(2^{\log_2 e}\right)^x$

$e^x = 2^{\log_2 e \, x}$

$y = 2^x \Rightarrow y = e^x$

$2^x \qquad y = 2^{(\log_2 e)x}$

Dilate factor $\frac{1}{\log_2 e}$ from the y axis