Exponential equations

Wednesday, 23 May 2018 5:59 pm

By the end of the teaching of this module I ask that the following work is completed:

Exponential Equations	5C	2,3(s),4(s)

RECAP:

In Year 10 and 11 you would have met and practiced how to use a number of Index Laws.
These are going to come in very handy now.

The laws you were shown were:

- $a^n \times a^m = a^{n+m}$
- $a^n \div a^m = a^{n-m}$
- $(a^x)^y = a^{xy}$
- $(ab)^x = a^x b^x$
- $\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$
- $a^{-x} = \frac{1}{a^x}$
- $a^0 = 1$

$x^2 \times x^3 = x^5$

$x^3 \div x^2 = x^1$

$(a^2)^3$ $a^2 \times a^2 \times a^2 = a^6$

$(a^2 b^3)^2 = a^4 b^6$

$(2ab^2)^3 = 8a^3b^6$

We can use these laws, and some common sense to help us solve some pretty complex questions.

Examples of exponential equations

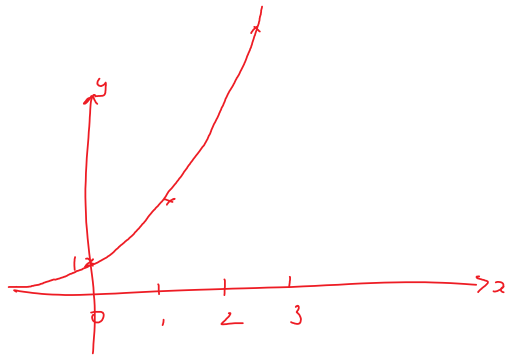
These are awesome!
They appear in Maths exams all the time.
In most cases we are simply looking to compare powers by ensuring the number first have the same base.

Example 1:
Find the value for x for which $5^x = 3125$

$5^x = 5^5$

$x = 5$

5
25
125
625
3125



Example 2:
Find the value for x for which $3^{x-1} = 243$

$3^{x-1} = 3^5$

$x - 1 = 5$

$\therefore x = 6$

3
9
27
81
243

$243 = 3^5$

Example 3:
Find the value for x for $5^{2x-4} = 25^{-x+2}$

$5^{2x-4} = 25^{-x+2}$

$5^{2x-4} = (5^2)^{-x+2}$

$5^{2x-4} = 5^{-2x+4}$

$2x - 4 = -2x + 4$

$4x - 4 = 4$

$25 = 5^2$

$4x = 8$

$x = 2$

Exam questions and then some!

The following types of questions appear on exams all the time.
They are a favourite of Mathematics teachers the world over as they combine two subject areas:

- Quadratics
- Powers

The trick is to noticing the pattern.

Remember back to exponential graphs where I told you that you need to look out for $y = 2 \times e^{3x}$

The multiplication sign here simply means that there is a dilation.
The form was what you were looking for to know how to proceed.

Here is the form of an quadratic disguised as an exponential:

Example 4:
Solve for x the equation $9^x = 12 \times 3^x - 27$

$9^x - 12 \times 3^x + 27 = 0$

$(3^x)^2 - 12 \times 3^x + 27 = 0$

$a = 3^x$ $\quad a^2 - 12a + 27 = 0$

$a^2 - 3a - 9a + 27 = 0$

$a(a-3) - 9(a-3) = 0$

$(a-3)(a-9) = 0$

$9^x = (3^2)^x \leftarrow 3^{2x}$

$= (3^x)^2$

27	
1	27
-3	-9

$\therefore$ $a - 3 = 0$ or $a - 9 = 0$

$a = 3$ or $a = 9$

$3^x = 3$ or $3^x = 9$

$x = 1$ $\qquad x = 2$

Example 5:
Solve for x the equation $3^{2x} = 27 - 6 \times 3^x$

$3^{2x} + 6 \times 3^x - 27 = 0$

$(3^x)^2 + 6 \times 3^x - 27 = 0$

$a = 3^x$ $\quad a^2 + 6a - 27 = 0$

$a^2 - 3a + 9a - 27 = 0$

$a(a-3) + 9(a-3) = 0$

$(a+9)(a-3) = 0$

-27	
-1	27
-3	9

$\therefore$ $a + 9 = 0$ $\qquad a - 3 = 0$

$a = -9$ $\qquad a = 3$

$\cancel{3^x = -9}$ $\qquad 3^x = 3$

No valid solution $\qquad x = 1$