Dilations

Sunday, 11 February 2018 7:34 pm

By the end of the lesson I would hope that you have an understanding of the concepts below which you can apply to a number of complex questions:

- Know what it means to dilate something
- Know how to use the correct notation
- Apply transformations to sketches of graphs
- Know how to use transformation algebra to go from a base graph to a transformed graph

RECAP:

In the previous lesson we looked at how we can translate ordered pairs, or a set of ordered pairs using either algebra, or a short cut.
Translations form just one part of a set of transformations.
This lesson will build on the basis and now introduce the idea of a dilation

WHAT IS A DILATION?

Don't you love a good dictionary definition!

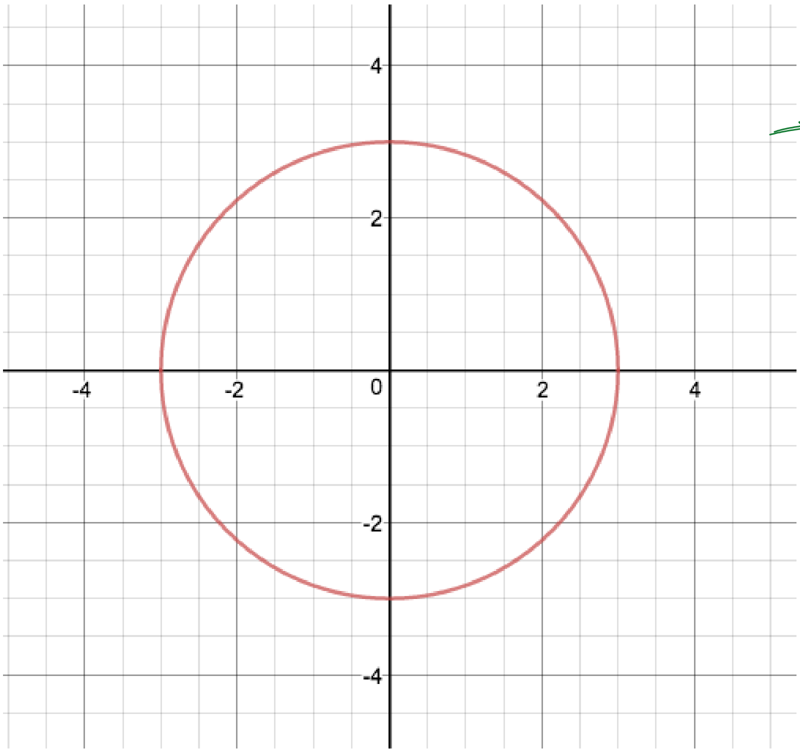
dilation
/dɪˈleɪʃ(ə)n,dʌɪˈleɪʃ(ə)n/

noun
noun: **dilation**; plural noun: **dilations**

1. PHYSIOLOGY
   the action or condition of becoming or being made wider, larger, or more open.
   "nitric oxide causes dilation of the blood vessels"
2. the action of speaking or writing at length on (a subject).
   "the main editorial involved no dilation on the privileges or responsibilities of citizenship"

Dilation is a stretching of the function.
This can be described as away from the x-axis or away from the y-axis.

Consider the following circle:

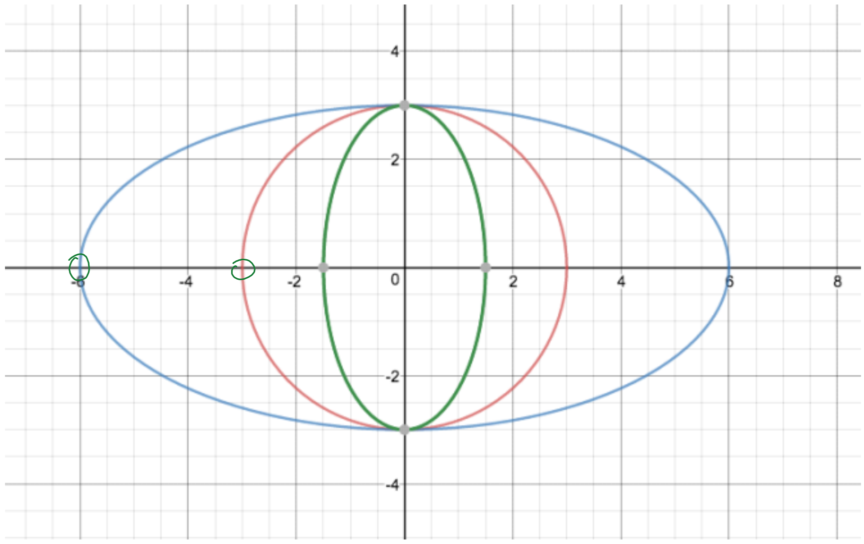


$x^2 + y^2 = 9$

Remember that the general formula for a circle is given by:
$x^2 + y^2 = r^2$

Hence the circle on the left has a centre of the origin with radius 3.

When we dilate away from the y-axis we end up with functions which look like:



$x^2+y^2=9$

$\left(\frac{x}{2}\right)^2+y^2=9$

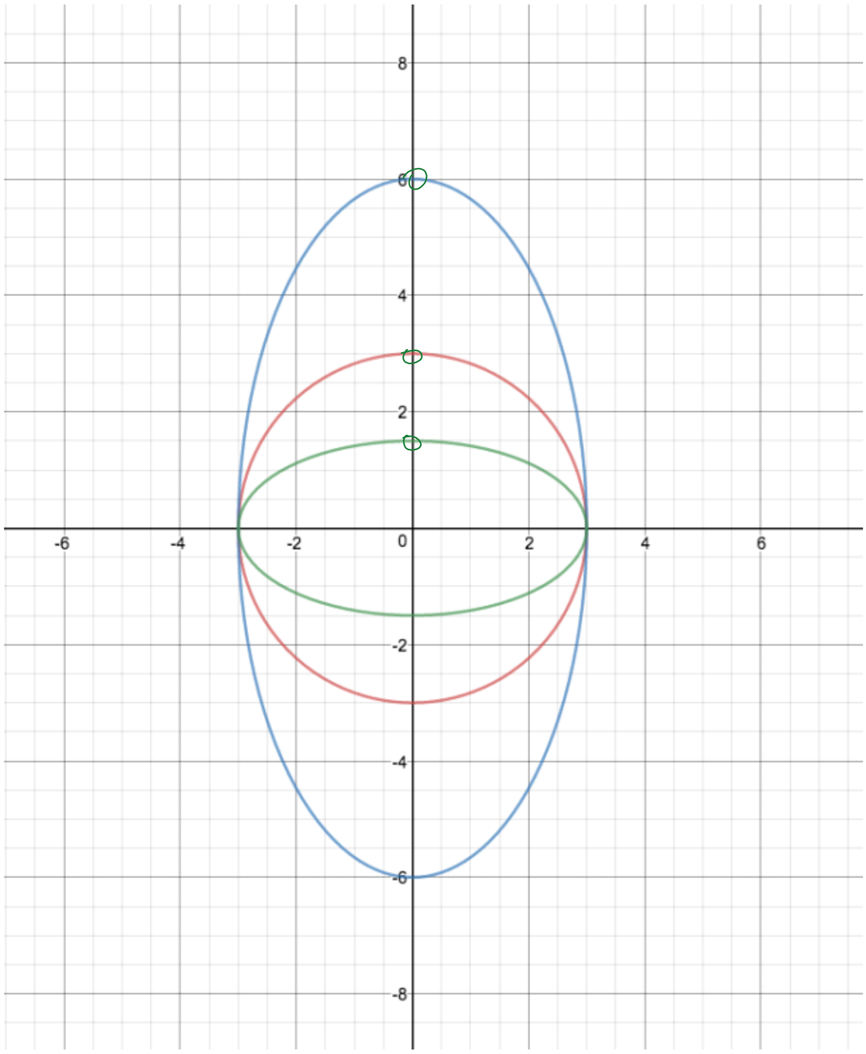
$(2x)^2+(y)^2=9$

$\frac{x}{2}$

$y=(x-1)^2$

When we dilate away from the y-axis we change the x-values

When we dilate away from the x-axis we end up with functions which look like:



$x^2+y^2=9$

$(x)^2+\left(\frac{y}{2}\right)^2=9$

$(x)^2+(2y)^2=9$

$y \to \frac{y}{2}$

When we dilate away from the x-axis we change the y-values

Dilations from the x- and y-axes using ALGEBRA

A coordinate and its image can be expressed using algebra: (x,y) maps onto (x',y')

Dilation from the x-axis

If we have a dilation of factor 2 away from the x-axis, we are effectively doubling all the y-values (x,y) maps onto $(x,2y)$

Dilation from the y-axis

If we have a dilation of factor 2 away from the x-axis, we are effectively doubling all the y-values

(x, y) maps onto $(x, 2y)$

Dilation from the y-axis

If we have a dilation of factor 2 away from the y-axis, we are effectively doubling all the y-values

(x, y) maps onto $(2x, y)$

Example using an ordered pair

$(2,-3)$ factor 2 $\uparrow x$ $(x,y) \rightarrow (x, 2y)$

$(2,-6)$

$(x,y) \rightarrow (3x, y)$

$(2,-3)$ for $3 \rightarrow y$

$(6,-3)$

Example using algebra:
Find the equation of a circle with centre the origin and radius 4, which has been subjected to a dilation from the x-axis of 2 units and a dilations form the y-axis of 3 units.

$x^2 + y^2 = 16$ $\uparrow \times 2$ $\xrightarrow{\times 3}$

$(x,y) \rightarrow (x, 2y) \rightarrow (3x, 2y)$

(x', y')

$3x = x'$ $2y = y'$

$x = \frac{x'}{3}$ $y = \frac{y'}{2}$

$x^2 + y^2 = 16$

$\left(\frac{x'}{3}\right)^2 + \left(\frac{y'}{2}\right)^2 = 16$

$\left(\frac{x}{3}\right)^2 + \left(\frac{y}{2}\right)^2 = 16$

Short cuts

Dilation from the x-axis factor b

$y = f(x)$

Where you see y, replace it $\frac{y}{b}$

$\frac{y}{b} = f(x)$

Dilation from the y-axis factor a

$y = f(x)$

Where you see x, replace it $\frac{x}{a}$

$y = f(\frac{x}{a})$

Example:
For the function, $f(x) = (x - 3)^2 + 4$, find the equation of the transformed function is $f(x)$ has been subjected to a dilations from the x-axis factor 2 and factor 4 from the y-axis.

$f(x) = (x-3)^2 + 4$ $\uparrow \times 2$ $\xrightarrow{\times 4}$ $\frac{x}{4}$ $\frac{y}{2}$

$\frac{y}{2} = \left(\frac{x}{4} - 3\right)^2 + 4$

$y = 2\left(\frac{x}{4} - 3\right)^2 + 8$

Doing it backwards

Sadly, we are not always good at teaching things in both directions!
What you can do one way you must, must, must be able to reverse.
This is the difference between regurgitating skills and understanding what you are doing.

Example:
Determine the factor of dilations when the graph of $y = \sqrt{4x}$ is obtained from the graph of $y = \sqrt{x}$
From:

- The y-axis
- The x-axis

$y = \sqrt{x} \rightarrow y = \sqrt{4x}$

$y = \sqrt{\underline{\underline{x}}}$ $\quad$ $y' = \sqrt{4\underline{\underline{x'}}}$

$x = 4x'$ $\quad$ $y = y'$ $\quad$ $x' = \frac{x}{4}$ $\quad$ $y' = y$

$(x, y) \longrightarrow (x', y')$

$(x, y) \longrightarrow \left(\frac{x}{4}, y\right)$

x-axis

$y = \sqrt{4x}$ $\quad$ $y = \sqrt{x}$

$y' = \sqrt{4}\sqrt{x'}$

$\frac{y'}{2} = \sqrt{x'}$ $\quad$ $y = \frac{y'}{2}$ $\quad$ $x = x'$

$y = \sqrt{x}$

$x' = x$ $\quad$ $y' = 2y$

$(x, y) \longrightarrow (x', y')$

$(x, y) \longrightarrow (x, 2y)$