Combinations of transformations

Tuesday, 13 February 2018 12:18 PM

By the end of the lesson I would hope that you have an understanding of the concepts below which you can apply to a number of complex questions:

- Know what it means to combine transformations
- Know the order of how to apply transformations (if an order hasn't already been given)
- Know how to apply the transformations to sketches of common functions

RECAP:

In our previous lessons we took a look at how to translate, dilate and reflect some pretty basic functions.
This was done using algebra (for the most part) and short cuts.
Whilst we only did things one at a time in the previous videos, we are now going to apply a number of transformations at the same time.

How do I know what order to apply transformations in?

There is a rule of thumb which tells you the order to apply transformations in.
It's called DR T.
It basically means:

- Dilations,
- Reflections and then
- Translations

When a question is not clear about the order it wants you to transform, you must use the order shown above.

In the examples below we will look at the questions where they given an order.

Short cut revision

Whilst I'm not to worlds greatest lover of remembering short cut after short cut … here is a summary of the ones from previous lessons. Note, I will be using algebra to show how to do the questions as well as short cuts and graphing!

Translations:

Where you see an 'x' in the function replace it with $x - h$.
Where you see a 'y', replace it with $y - k$

Dilations:

Dilation from the x-axis factor b

$y = f(x)$ Where you see y, replace it $\frac{y}{b}$

$\frac{y}{b} = f(x)$

Dilation from the y-axis factor a

$y = f(x)$ Where you see x, replace it $\frac{x}{a}$

$y = f(\frac{x}{a})$

Reflections:

Reflection in the x-axis e.g. $y = f(x) becomes \ -y = f(x)$ Replace y with $-y$ in the equation.

Reflection in the y-axis e.g. $y = f(x) becomes \ y = f(-x)$ Replace x with -x in the equation

Let's get to work with some questions …

Example 1:
Find the equation of the image of $y = x^2$ under a dilation of factor 2 from the x-axis, followed by a translation 3 units in the positive

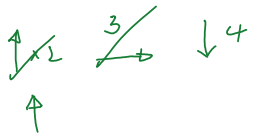
direction of the x-axis and 4 units in the negative direction of the y-axis, followed by a reflection in the x-axis.

It can be done in one of three ways:

- Using algebra
- Using "short cuts"
- Drawing successive graphs

The method you use will depend on what the question is asking of you, and what information they give you at the beginning.

Solution: Using Algebra



$(x, y) \rightarrow (x, 2y) \rightarrow (x+3, 2y) \rightarrow (x+3, 2y-4)$

$(x : y')$

$x' = x + 3 \qquad y' = 2y - 4$

$x = x' - 3 \qquad y' + 4 = 2y$

$y = \frac{y' + 4}{2}$

$y = x^2$

$\frac{y' + 4}{2} = (x' - 3)^2$

$y + 4 = 2(x - 3)^2$

$y = 2(x - 3)^2 - 4$

Solution: Using shortcuts

a dilation of factor 2 from the x-axis, followed by a translation 3 units in the positive direction of the x-axis and 4 units in the negative direction of the y-axis, followed by a reflection in the x-axis.

$y = x^2 \rightarrow 3 \downarrow 4$

$\uparrow \times 2$

$\frac{y}{2} = x^2$

$y = 2x^2$

$y = 2(x - 3)^2$

$y + 4 = 2(x - 3)^2$

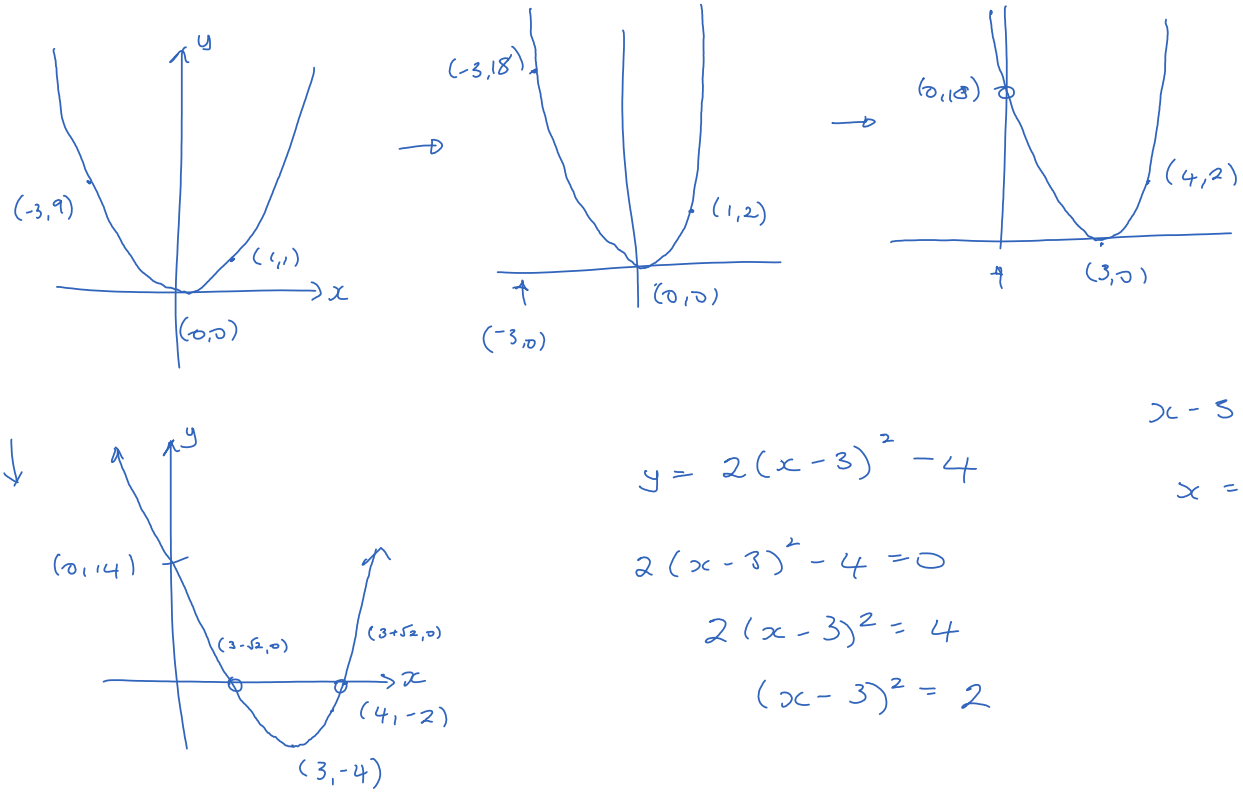
$\therefore y = 2(x - 3)^2 - 4$

Solution using graphical means

a dilation of factor 2 from the x-axis, followed by a translation 3 units in the positive direction of the x-axis and 4 units in the negative direction of the y-axis, followed by a reflection in the x-axis.

When drawing graphs (at any point in Methods), you must always ensure to label the following:

- x-axis intercept(s) **in co-ordinate form**
- y-axis intercept(s) **in co-ordinate form**
- Turning points (maxima and minima)
- Endpoints (when the domain is limited)
- Equations of any asymptotes
- At least one co-ordinate value



$y = 2(x-3)^2 - 4$

$2(x-3)^2 - 4 = 0$

$2(x-3)^2 = 4$

$(x-3)^2 = 2$

$x - 3 = \pm\sqrt{2}$

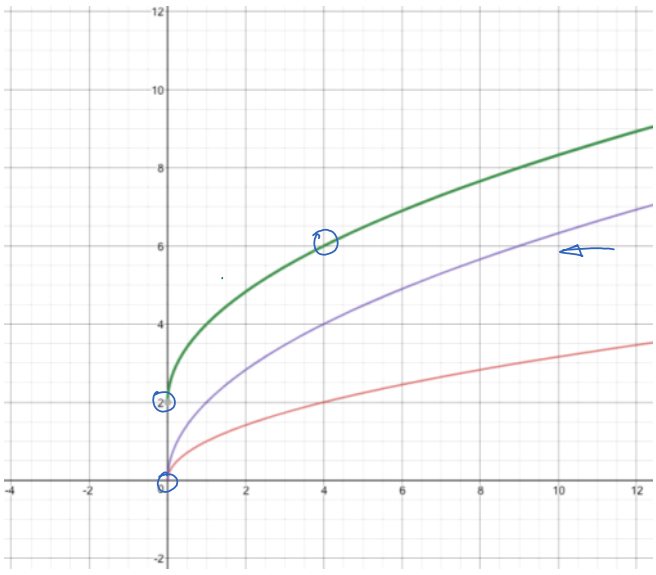
$x = 3 \pm \sqrt{2}$

When the order isn't done correctly:

Remember: The order of transformations is important.
When you're not given the order you do DrT.

Here is an example of a graph where we dilate by 2 from the x-axis and translate 2 in the positive direction of the y-axis but in a different order.

$\sqrt{x}$



1. $y = \sqrt{x}$
2. Dilate first ...
3. $y = 2\sqrt{x}$
4. Then translate ...
5. $y = 2\sqrt{x} + 2$

1 $y = \sqrt{x}$

2 Translate first ...

3 $y = \sqrt{x} + 2$

4 Then dilate ...

5 $y = 2(\sqrt{x} + 2)$