What is a matrix?

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By the end of the lesson I would hope that you have an understanding and be able to apply to questions the following concepts:

- What a matrix is
- What a matrix is used for
- Understand the following matrix notation
    - Row
    - Column
    - Order
    - Row matrices
    - Column matrices
- Know how to transpose a matrix
- Know what a square matrix is
- Know what diagonal, symmetric and triangular matrices are
- Know what the identity matrix is for and how to use it
- Know what a triangular matrix is
- How to use the CAS to manipulate matrices

RECAP

This is the first lesson in this series on Matrices.
This is a Module for the Further Mathematics Units 3 and 4 course but is universal in it's application (hence it's useful to all courses all over the world).

The Matrix?

Sadly, we're not going to be dealing with the movie!
I had absolutely no idea what was going on in that whole franchise!
Thankfully I don't have to teach it!!!

How does your credit and debit card work?

This is how I understand it!

Inside the little chip which you either tap (or insert into the hole in the wall) is a very large mathematical object called a matrix.

When you tap, the card reader does some lightening fast Maths using this Matrix and sends a code to the main servers to see if the card is valid and if you have enough money to pay for something.

It's not a password. It's not really your pin (although that's in there too). It's a massive, two dimensional set of numbers.

A Mathematical Matrix

This is an example of a matrix:

$$G = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

You can see a number of things.

1. It's square (matrices don't have to be square!]
2. It has numbers which fit into both rows and columns.
3. The numbers are encased in square brackets
4. Notice the letter in front of the matrix.

Let's have a little bit of order!

Matrices are **defined** by their rows and columns.
It's how we name them.
The name we give is called the order.

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

This is a row.

The matrix has three rows

This is a column

The matrix has three columns

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

The **order of a matrix** is described as:

$$(\,rows \times columns)$$

$$(\,rows\ by\ columns)$$

The following Matrix would hence be described as a 3 by 3 or 3 $\times$ 3

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

3 by 3

3 x 3

Other examples:

$$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$$

1 x 3 1 by 3

$$\begin{bmatrix} 5 \\ 6 \\ 7 \\ 8 \end{bmatrix}$$

4 x 1 4 by 1

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$$

3 x 2 3 by 2

Row matrices

As the name suggests, these are matrices which have a single row.

$$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$$

Columns matrices

As the name suggests, these are matrices which have a single column.

$$\begin{bmatrix} 5 \\ 6 \\ 7 \\ 8 \end{bmatrix}$$

Transposing matrices

In my job as Head of Maths I deal with a lot of data.
There are times when, using Microsoft Excel, I wish to turn data which is in a row, and list it as a column.
This process is called **transposing a matrix.**

To **transpose** a matrix you swap the rows and columns,

$$\underset{3 \times 2}{\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}^T} \longrightarrow \underset{2 \times 3}{\begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}}$$

Notice the floating T to say that the transpose of a matrix becomes.

Diagonal Matrices

Matrices have two diagonals!
The one highlighted below is probably the more useful one and hence it's called the leading diagonal.
The following example is NOT a diagonal matrix!

$$\begin{bmatrix} 1 & 3 & 5 & 7 \\ 2 & 4 & 6 & 8 \\ 4 & 3 & 2 & 1 \\ 9 & 8 & 7 & 6 \end{bmatrix} \leftarrow$$

This is an example of a diagonal matrix:
Note: It must be square and have all numbers outside of the leading diagonal as zeros.

Note: It must be square and have all numbers outside of the leading diagonal as zeros.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 6 \end{bmatrix}$$

Identity matrices

Identity matrices are a specific type of diagonal matrix!
All the numbers in the leading diagonal are one's.
Identity matrices must also be square!

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

1×1

$\begin{bmatrix} 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

2×2 3×3

Symmetric matrices

This is a matrix which, when transposed, looks exactly the same.
The best looking people are, apparently, those with symmetrical faces which means the left hand side and the right hand side are pretty much the same.

Examples of symmetric matrices are shown below:

$$\begin{bmatrix} 4 & 5 \\ 5 & 3 \end{bmatrix} \qquad \begin{bmatrix} 2 & 3 & 4 \\ 3 & 1 & 5 \\ 4 & 5 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 5 \\ 5 & 3 \end{bmatrix} \qquad \begin{bmatrix} 2 & 3 & 4 \\ 3 & 1 & 5 \\ 4 & 5 & 3 \end{bmatrix}$$

Triangular matrices

These come in two types; upper and lower triangular matrices.
Examples are shown below

$$\begin{bmatrix} 4 & 5 & 6 \\ 0 & 3 & 4 \\ 0 & 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 4 & 0 & 0 \\ 5 & 3 & 0 \\ 6 & 7 & 1 \end{bmatrix}$$

UPPER LOWER

Barry has been at it again!!!

Maths would be great if we just kept it out of the hands of Barry.
But here is a prime example of how he tries and tricks us!

Each element in a matrix has a code.
It helps us talk about individual numbers

$$A = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 2 & 4 & 6 & 8 \\ 4 & 3 & 2 & 1 \\ 9 & 8 & 7 & 6 \end{bmatrix}$$

a_{23}

a_{23}

a_{41}

The subscript numbers are really just a cell reference (much like Microsoft Excel).
The format is:

$$a_{row\ column}$$

I've left the space in for readability!
Notice the difference between upper case and lower case letters used!

Let's do some examples ...

Example

The following example has been extracted, with permission, from the Cambridge Further Maths Units 3 and 4 Textbook.

Write down the transpose of $\begin{bmatrix} 7 & 4 \\ 8 & 1 \end{bmatrix}$

$$\begin{bmatrix} 7 & 8 \\ 4 & 1 \end{bmatrix}$$

Write down the matrix $\begin{bmatrix} 0 & 1 & 2 \\ 3 & 2 & 5 \end{bmatrix}^T$

$$\begin{bmatrix} 0 & 3 \\ 1 & 2 \\ 2 & 5 \end{bmatrix}$$

If $A = [2 \quad 10 \quad 1]$ write does the matrix A^T

$$\begin{bmatrix} 2 \\ 10 \\ 1 \end{bmatrix}$$

Example

The following example has been extracted, with permission, from the Cambridge Further Maths Units 3 and 4 Textbook.

For each of the matrices below, write down its type, order and the number of elements.

$A = \begin{bmatrix} 1 & 5 & 1 \\ 2 & 2 & 4 \\ 2 & -1 & 6 \end{bmatrix}$ Square, 3 by 3, 9

$B = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ Column, 3 by 1, 3

$$B = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$ Column, 3 by 1, 3

$$C = \begin{bmatrix} 3 & 1 & 0 & 5 & -3 & 1 \end{bmatrix}$$ Row, 1 x 6, 6

Example

The following example has been extracted, with permission, from the Cambridge Further Maths Units 3 and 4 Textbook.

Consider the following square matrices.

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 3 \end{bmatrix} \quad \begin{bmatrix} 1 & 3 & 5 \\ 3 & 4 & 7 \\ 5 & 7 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & 5 \\ 3 & 4 & 7 \\ 5 & 7 & 2 \end{bmatrix}$$

Identity Upper Δ Diagonal Sym

Write down:

a. the upper triangular matrices
b. the identity matrix
c. the diagonal matrices
d. the symmetric matrices.

Example

The following example has been extracted, with permission, from the Cambridge Further Maths Units 3 and 4 Textbook.

For the matrices A and B, opposite, write down the values of:

$$A = \begin{bmatrix} 1 & 5 & 3 \\ -1 & 0 & 4 \\ 2 & -2 & 6 \end{bmatrix} \quad B = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

a_{12}
a_{21}
a_{33}
b_{31}

$a_{12} = 5$

$a_{21} = -1$

$a_{33} = 6$

$b_{31} = 1$

Example

The following example has been extracted, with permission, from the Cambridge Further Maths Units 3 and 4 Textbook.

A is a 2×2 matrix. The element in row i and column j is given by $a_{ij} = i + j$. Construct the matrix.

This is a really important question and comes up in the exam all the time!

$$\begin{array}{cc} & \begin{array}{cc} 1 & 2 \end{array} \\ \begin{array}{c} 1 \\ 2 \end{array} & \begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix} \end{array}$$

a_{ij}

row column

$a_{ij} = i + j$

row + column

Using your CAS

Your CAS can help with Matrices!

Enter the matrix $A = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 4 & 2 \end{bmatrix}$ and determine its transpose (A^T).