Volumes and surface areas

Sunday, 20 October 2019 11:41 am

By the end of the lesson I would hope that you have an understanding (and be able to apply to questions) the following concepts:

- Know what it means by the word volume
- Know what it means by the words surface area
- Know how to find the volume and surface areas of the following shapes
  - Cylinder
  - Pyramid
  - Sphere
  - Cone
  - Composite shapes
  - Cuboid

RECAP

This series of lessons has been looking at the fascinating topic of geometry and measurement. We've covered a lot of ground so far … but how is any of this useful to me? Let's take a look at some of the more interesting uses of Mathematics!

Pythagoras' Theorem, Parallel Lines

Builders use this all the time to make sure that corners of house are right angled!

DIY uses a LOT of Mathematics

When I paint the walls of my house, I need to go to Bunnings and purchase paint.
How do I know how much paint I need to buy?
Use Mathematics!

Walls have a surface area.

When I wish to landscape my garden and place some pebbles all along the borders, I need to use VOLUME to be able to order the correct amount of pebbles.

Knowing how to find the volumes and surface area of common shapes actually is some of the most important part of Mathematics you might ever learn.

Volumes of shapes

Let's look at the volumes of some shapes starting with a prism

prism
/ˈprɪz(ə)m/

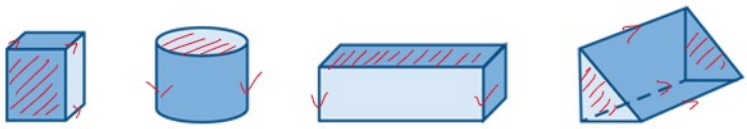
noun

GEOMETRY
a solid geometric figure whose two ends are similar, equal, and parallel rectilinear figures, and whose sides are parallelograms.

- OPTICS
  a glass or other transparent object in the form of a prism, especially one that is triangular with refracting surfaces at an acute angle with each other and that separates white light into a spectrum of colours.
- used to refer to the clarification or distortion afforded by a particular viewpoint.
  "they were forced to imagine the disaster through **the prism of** television"

I like to think of a prism as much like a loaf of bread!

Here are some examples of prisms:

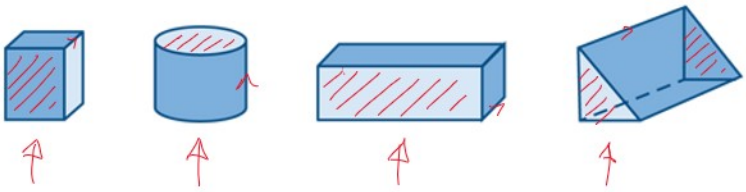


We can find the volumes of prisms using a simple rule:

$$Volume = Area\ of\ cross\ section \times height$$

Area

The cross section is the face which, when you slice the shape, all slices will have the same shape.



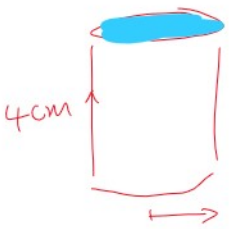
Units of VOLUME

Remember that the units of volume always have a **floaty three**. I like to think that this is true because it's three dimensional. So, units will be in mm^3, cm^3, m^3, km^3 etc

Examples

The following examples are used, with permission, from the Cambridge Further Mathematics Units 3 and 4 textbook.

Example 1
Find the volume of a cylinder, which has radius 3 cm and height 4 cm, correct to two decimal places.



Vol = (Area of XS) x height
= $\pi r^2 \times h$

$= \pi r^2 \times h$

$= \pi \times 3^2 \times 4$

$= 113.10\ cm^3$

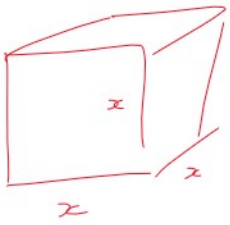
Finding the volumes of common prisms

Here are the formulae for the most common prisms:

Cylinder

$V = \pi r^2 \times h$

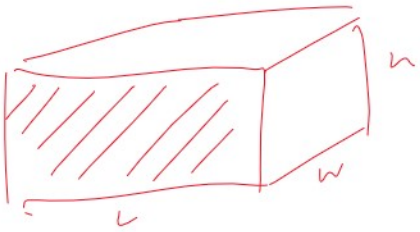
Cube



$Vol = x \times x \times x$

$= x^3$

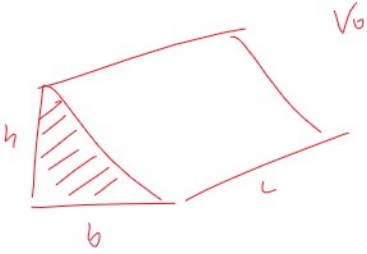
Rectangular prisms



$V = L \times h \times w$

$= LWh$

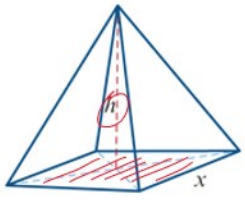
Triangular prism



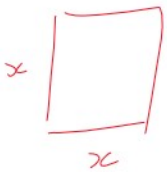
$Vol = \frac{bh}{2} \times L$

Volume of a Pyramid

A pyramid is not a prism!
It is a shape we often want to find the volume of though.
The volume of a **right pyramid** is:



~~$Volume = \frac{1}{3} \times base\ area \times height$~~

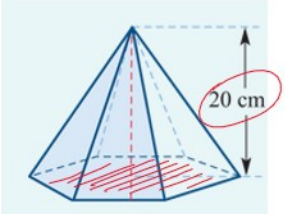


$Vol = \frac{1}{3} \times x \times x \times h$

$= \frac{1}{3} x^2 h$

Example 2
Find the volume of this hexagonal pyramid with a base area of 40 cm^2 and a height of 20 cm. Give the answer correct to one decimal place.

Find the volume of this hexagonal pyramid with a base area of 40 cm^2 and a height of 20 cm. Give the answer correct to one decimal place.



40 cm^2

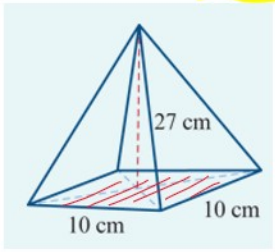
$$Vol = \frac{1}{3} \times BA \times h$$

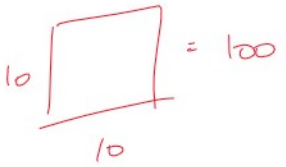
$$= \frac{1}{3} \times 40 \times 20$$

$$= 266.7\ cm^3$$

Example 3

Find the volume of this square pyramid with a square base with each edge 10 cm and a height of 27 cm.





$$Vol = \frac{1}{3} \times BA \times h$$

$$= \frac{1}{3} \times 100 \times 27$$

$$= 900\ cm^3$$

Volume of a cone

If I were to sell ice-cream, then I'd want to make sure I knew the volume of ice-cream that was going to fit into each cone to make sure I was maximising profit!

The volume of a cone is:

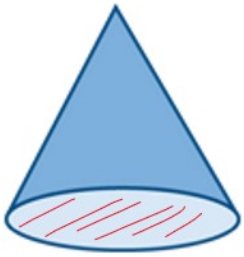
$$\frac{1}{3} \times BA \times h$$

$$Volume = \frac{1}{3} \times \pi \times r^2 \times height$$

The volume of a cone is:

$\frac{1}{3} \times BA \times h$

$$Volume = \frac{1}{3} \times \pi \times r^2 \times height$$



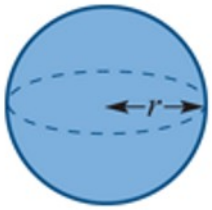
πr^2

Volume and surface are of a sphere

I want to make circular Easter eggs!
I need to know the volume of the eggs and the surface area to make sure I don't under fill it with chocolatey goodness and that I have enough gold foil to be able to wrap it properly.

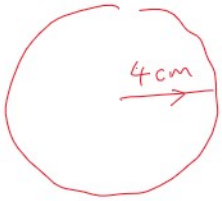
The volume and surface area found using the following formulae:

$$Volume = \frac{4}{3} \times \pi \times r^3$$



$$Surface\ area = 4 \times \pi \times r^2$$

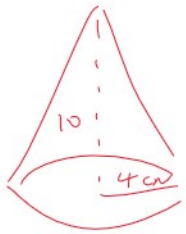
Example 4
Find the volume of a sphere with radius 4 cm and a cone with radius 4 cm and height 10 cm.



$$Vol = \frac{4}{3} \times \pi \times r^3$$

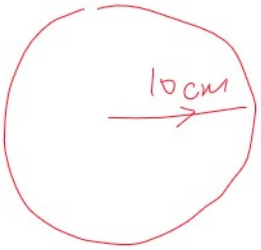
$$= \frac{4}{3} \times \pi \times 4^3$$

$$= 268.08\ cm^3$$



$$\text{Vol} = \frac{1}{3} \times BA \times h$$
$$= \frac{1}{3} \times \pi \times r^2 \times h$$
$$= \frac{1}{3} \times \pi \times 4^2 \times 10$$
$$= 167.55\ cm^3$$

Example 5
Find the surface area of a sphere with radius $10\ cm$.

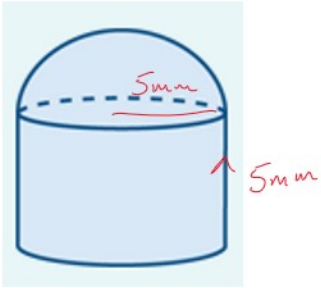


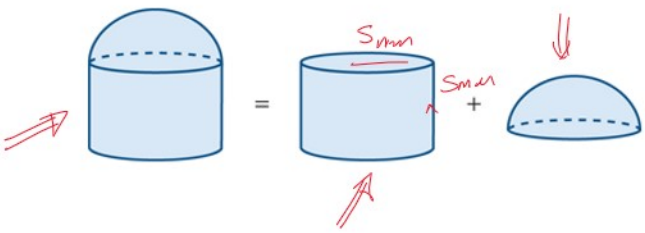
$$SA = 4 \times \pi \times r^2$$
$$= 4 \times \pi \times (10)^2$$
$$= 1256.64\ cm^2$$

Composite shapes

Composite shapes are when we put more than one of the "base" shapes together to form a bigger shape. When we work out the volume of composite shapes, we simply find the volumes of each smaller shape and then add them together.

Example 6
A hemisphere is placed on top of a cylinder to form a capsule. The radius of both the hemisphere and the cylinder is $5\ mm$. The height of the cylinder is also $5\ mm$. What is the volume of the composite soli in cubic millimetres, correct to two decimal places?





$V = \pi r^2 \times h$ $\quad V = \frac{4}{3} \times \pi \times r^3$

$= \pi \times 5^2 \times 5$ $\quad + \quad = \frac{4}{3} \times \pi \times 5^3 \times \frac{1}{2}$

$V = \pi \times 5^2 \times 5 + \frac{4}{3} \times \pi \times 5^3 \times \frac{1}{2}$

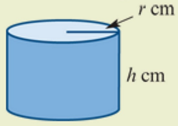
$= 654.50 mm^3$

Surface area of three-dimensional shapes

Finding the surface area of shapes can be really important.
To make it easier, the formulae to help you to do this are provided below.
The screen shot has been provided, with permission, from Cambridge Further Mathematics Units 3 and 4 Textbook

Surface area of solids

Cylinder
(radius r cm, height h cm)

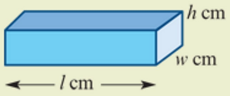


$S = 2\pi r^2 + 2\pi rh$

Cube
(all edges x cm)

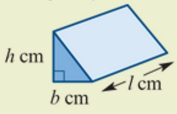
$S = 6x^2$

Rectangular prism
(length l cm), width w cm, height h cm)



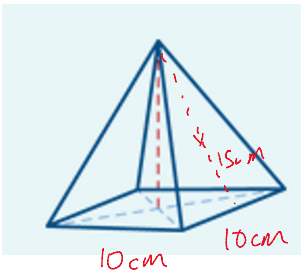
$S = 2(lw + lh + wh)$

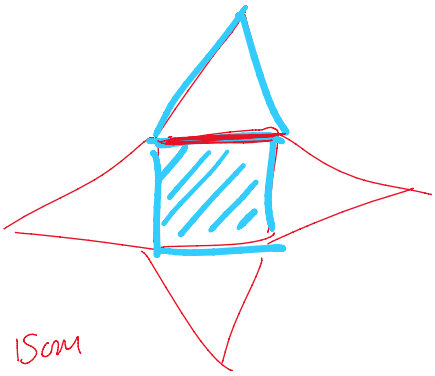
Triangular prism

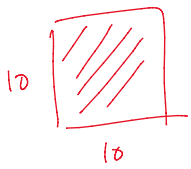


$S = bh + bl + hl + l\sqrt{b^2 + h^2}$

Example 7
Find the surface of the right square pyramid shown if the square base has each edge 10 cm in length and the isosceles triangles each have height 15 cm.







$$SA = 10 \times 10 + \left(\frac{1}{2} \times 10 \times 15\right) \times 4$$

$$= 400\ cm^2$$