Using Pythagoras' theorem in spheres

★ By the end of the lesson I would hope that you have an understanding and be able to apply to questions the following concepts:

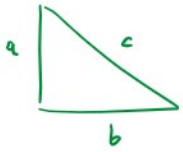
- How to use Pythagoras' Theorem in spherical problems

RECAP

This is the final lesson relating to spherical geometry and takes a look at how to apply Pythagoras' Theorem to spheres and hemispheres.

Remember, Pythagoras' Theorem relates to right angled triangles.

$$c^2 = a^2 + b^2$$



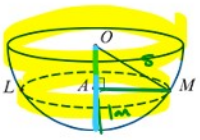
Example of how to apply

The following examples have been taken, with permission, from the Cambridge Further Mathematics Units 3 and 4 textbook.

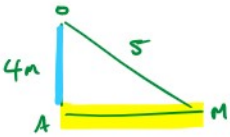
A pond in the shape of a hemisphere has radius 5 m contains water of depth 1 m.

Find:

- the length of the water line
- the area of the surface of the water
- the shortest distance along the surface of the bowl between L and M where LM is a diameter of the water surface (use the great circle through L and M).











- $C = 2 \times \pi \times r$

  $C = 2 \times \pi \times 3$

  $= 18.85m$

- $A = \pi r^2$

  $= \pi \times 3^2$

  $= 28.27m^2$

$5^2 = 4^2 + AM^2$

$25 = 16 + AM^2$

$AM^2 = 25 - 16$

$AM^2 = 9$

$AM = 3$

$\sin\theta = \frac{3}{5}$

$\theta = \sin^{-1}\left(\frac{3}{5}\right)$

$\theta = 36.87°$



$\text{Length} = \frac{73.74}{360} \times 2.\pi.5$

$= 6.44\ M$