The reciprocal transformation

Thursday, 23 January 2020 8:17 pm

By the end of the lesson I would hope that you have an understanding and be able to apply to questions the following concepts:

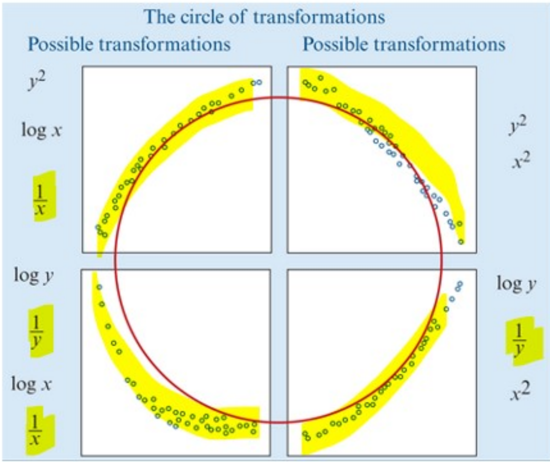
- How to apply the reciprocal transformation
- Why we need to apply the reciprocal transform

RECAP:

This lesson builds on the work we need to turn curves into lines.
In the last lesson we looked at how to use the log transformation to turn data from a curve into more linear. This was then used to find the least squares line to allow us to predict values.

This lesson is going to look at another transform, the reciprocal transform.

Remember, the circle of transformations will help us choose the most appropriate transform.



$\frac{1}{x}$

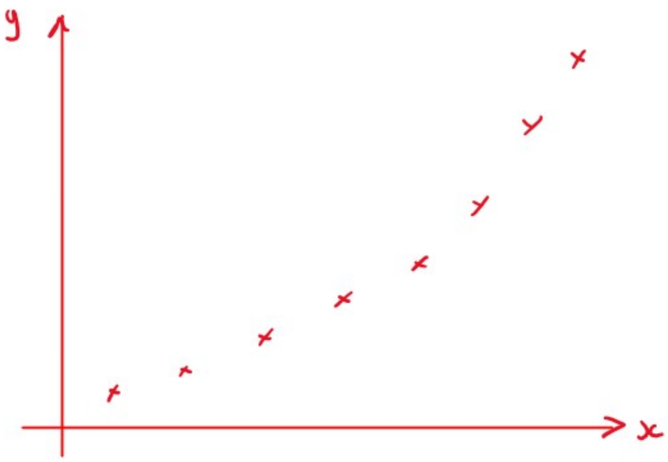
$\frac{1}{y}$

$1 \div x$

$1 \div y$

The reciprocal transform

It can be that our data looks like the following:



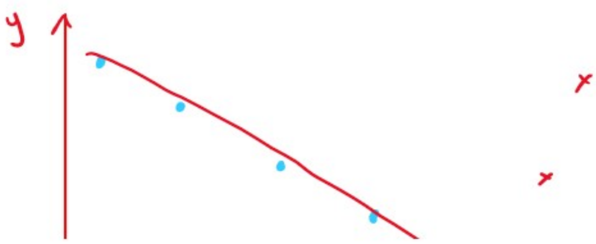
$\frac{1}{y}$

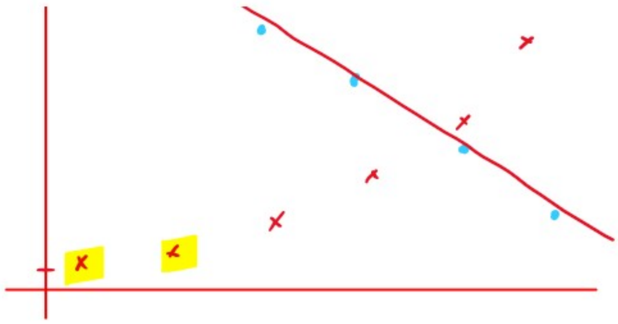
$\frac{10}{1} \rightarrow \frac{1}{10}$

$\frac{100}{1} \rightarrow \frac{1}{100}$

$\frac{2}{3} \rightarrow \frac{3}{2}$

It would be great then if we could keep the x values the same but change the y values to turn the data linear. Like so:





As it turns out, there is a way and it's called using the **reciprocal** transform.
When we take the reciprocal of a function we simply place it as the denominator of a fraction with the numerator as one.

Again, you need to remember that the equation of a straight line (or the least squares line) must be written correctly.

Label of the x-axis

Label of the y-axis

$$y = a + bx$$

y-axis intercept

Gradient of the line

$y = mx + c$

$y = a + bx$

It's **vitally important** that you realise the labels of the x- (and or y-) axes will now change.

Hence, for the example above, we would have an equation in the form:

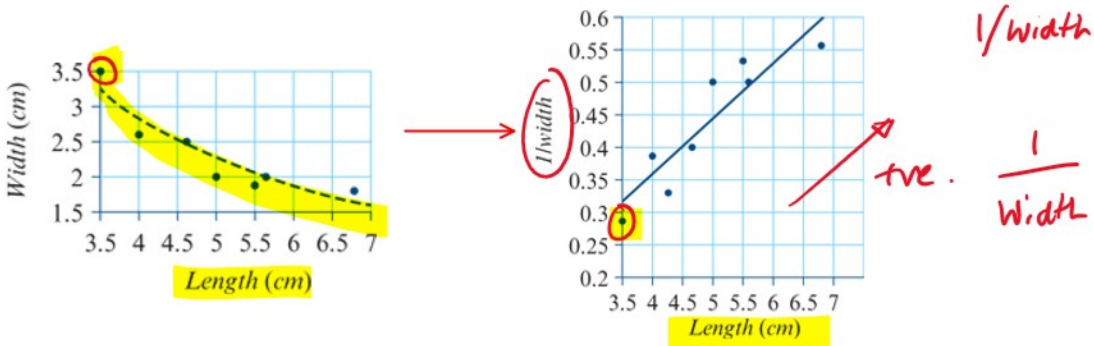
$y = a + bx$

$$y = a + b \times \frac{1}{x}$$

$\frac{1}{\text{Time}}$

The x-axis is now showing all the values of $\frac{1}{x}$

For a real world example:
Just to try and confuse you, here we are going to use a $\frac{1}{y}$ transformation!
This is clear from the label of the y axis on the newly transformed graph.



Hence, the equation of the least squares line is shown below.

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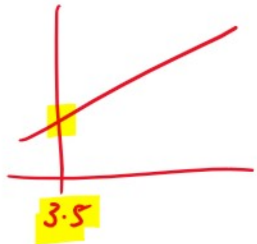
Note: This will make prediction calculations that little bit more **interesting!**

$y = a + bx$



$$\frac{1}{width} = 0.015 + 0.086 \times length$$

An example using the CAS

Remember, we can use the CAS in this whole course.
We are used to using the Statistics screen to enter values.
We will enter the raw data items we have and then get the calculator to change the y-values and then plot the appropriate graph.

The following example has been extracted with permission from the Cambridge Further Maths Units 3 and 4 textbook.

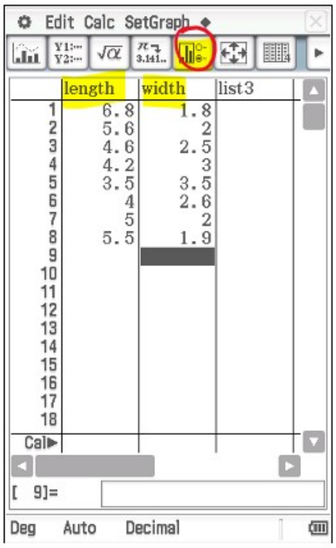
The table shows the length (in cm) and width (in cm) of eight sizes sticky labels.

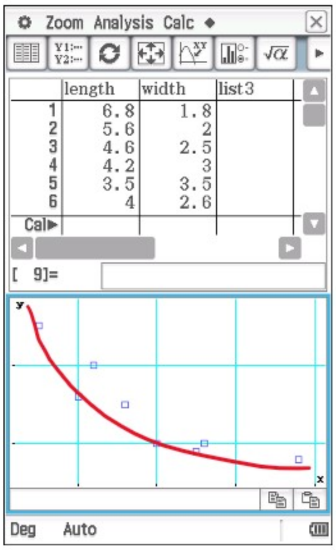
Length	6.8	5.6	4.6	4.2	3.5	4.0	5.0	5.5
Width	1.8	2.0	2.5	3.0	3.5	2.6	2.0	1.9

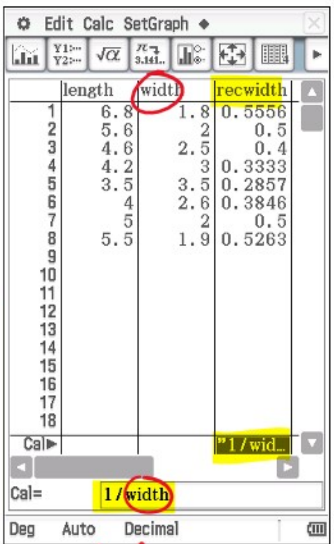
Using the $1/y$ transformation:

- linearise the data, and fit a regression line to the transformed data. Length is the EV.
- write its equation in terms of the variables length and width.
- use the equation to predict the width of a sticky label with length of 5 cm.

Length is the EV!



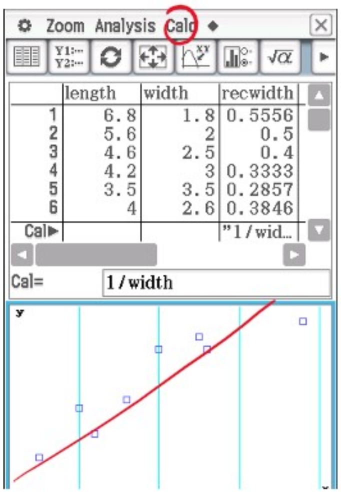


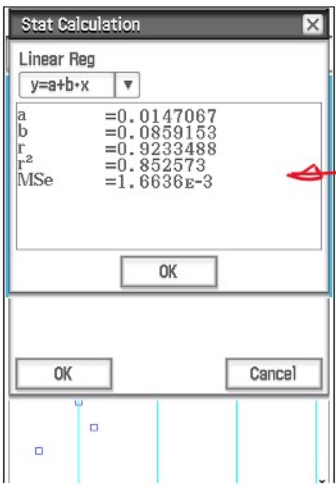


recw

w

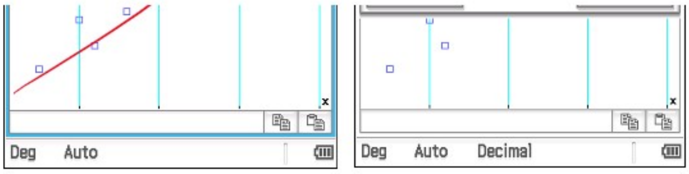
c





a = 0.0147

b = 0.086



Hence, using the information above, we can say that our least squares regression line is:

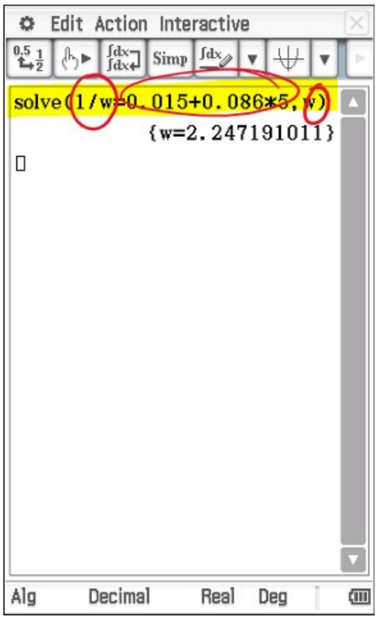
$\frac{1}{y}$

$$\frac{1}{width} = 0.015 + 0.086 \times height$$

Using the formula we can now find the width for the given length of 5cm

$$\frac{1}{width} = 0.015 + 0.086 \times height$$

$$\frac{1}{width} = 0.015 + 0.086 \times 5$$



To make life easier for myself I have used the SOLVE function of the CAS.

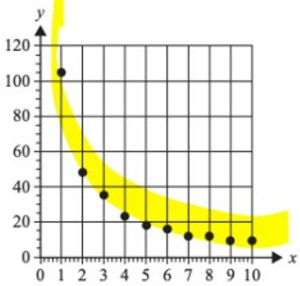
$$width = 2.25\ cm$$

VCAA Exam Question on this concept
2018 Paper 1

Question 11

Freya uses the following data to generate the scatterplot below.

x	1	2	3	4	5	6	7	8	9	10
y	105	48	35	23	18	16	12	12	9	9



The scatterplot shows that the data is non-linear.
To linearise the data, Freya applies a reciprocal transformation to the variable y.
She then fits a least squares line to the transformed data.
With x as the explanatory variable, the equation of this least squares line is closest to

A. $\frac{1}{y} = -0.0039 + 0.012x$

B. $\frac{1}{y} = -0.025 + 1.1x$

C. $\frac{1}{y} = 7.8 - 0.082x$

D. $\cancel{y = 45.3 + 59.7 \times \frac{1}{x}}$

E. $\cancel{y = 59.7 + 45.3 \times \frac{1}{x}}$

a = -3.95E-3

$\frac{1}{y}$

a = - 0.00395

b = 0.0118