The inverse matrix

Thursday, 23 April 2020 7:36 PM

★ By the end of the lesson I would hope that you have an understanding and be able to apply to questions the following concepts:

- Know what the inverse matrix is
- Know how to find the inverse matrix (and the determinant)
- How to find the inverse of a 2x2 matrix

RECAP

This is the start of a new section on Matrices.
We have looked at Matrices 1 - which was effectively all the theory behind how to manipulate matrices (with some applications!). We continue to look at how to use Matrices in Further Mathematics.

What can't we do with Matrices?

If you remember from the previous lessons, we can add, subtract and multiply matrices but we cannot divide them. So, we needed to come up with a way of dividing matrices.

Welcome to the inverse of a matrix

Sometimes, it's just not necessary to know why.
I don't know why the sky is blue.
I don't know why mosquitoes are needed.
I don't know why the inverse of a matrix is:

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$If\ A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\ then$$

$$A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Notice the notation for an inverse.

Simple examples of finding the inverse.

The following examples have been extracted, with permission from the Cambridge Further Maths Units 3 and 4 textbook.

Find the inverse of the following matrices:

$$A = \begin{bmatrix} 2 & 2 \\ 3 & 4 \end{bmatrix}$$

$$\begin{bmatrix} a & b \end{bmatrix}$$

$$A^{-1} = \frac{1}{8-6}\begin{bmatrix} 4 & -2 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -3 & 1 \end{bmatrix}$$

$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$\frac{1}{8-6}\begin{bmatrix} & \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} & \\ -\frac{3}{2} & 1 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 4 & -2 \\ -3 & 2 \end{bmatrix}$$

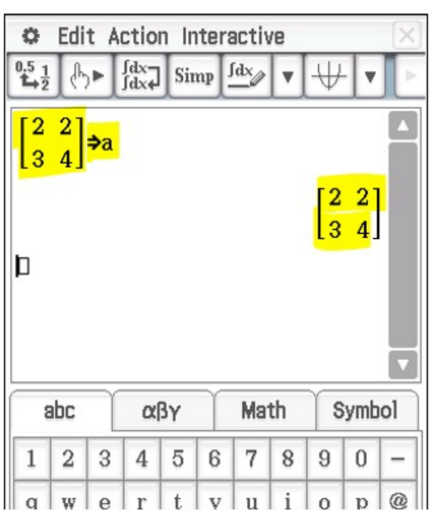
$A = \begin{bmatrix} 2 & 3 \\ 2 & 3 \end{bmatrix}$

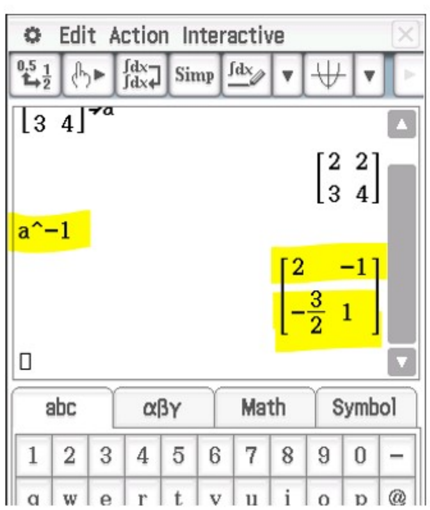
$$A^{-1} = \frac{1}{6-6}\begin{bmatrix} 3 & -3 \\ -2 & 2 \end{bmatrix}$$

$$= \cancel{\frac{1}{0}\begin{bmatrix} 3 & -3 \\ -2 & 2 \end{bmatrix}}$$

Using the CAS to find the inverse of a Matrix

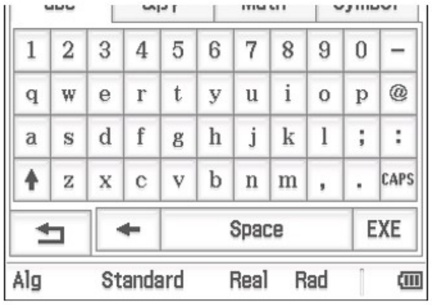
This is super simple!

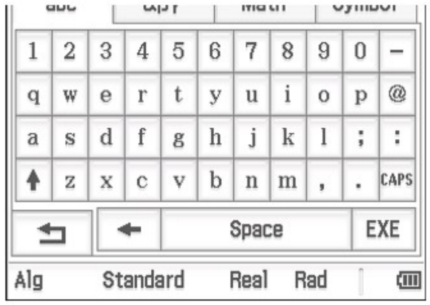




a^{-1}

a ^ -1





The determinant of a matrix

The determinant is a very useful thing when trying to find solutions to simultaneous equations (and is used a lot in Mathematical Methods!).

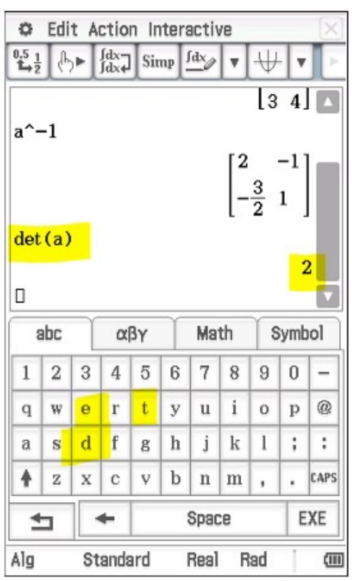
What is the determinant?

$$If\ A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\ then$$

$$\det(A) = ad - bc$$

It's the fraction bit in front of the inverse.

Your CAS can work this bit out for you too!



The determinant helps us find out if there are none, one or infinity many solutions when trying to solve simultaneous equations!

Examples of finding the determinant

The following examples have been extracted, with permission from the Cambridge Further Maths Units 3 *and 4 textbook.*

Find the determinant of the matrices:

$A = \begin{bmatrix} 2 & 3 \\ 3 & 5 \end{bmatrix}$

$$\det(A) = 2 \times 5 - 3 \times 3$$
$$= 10 - 9$$
$$= 1$$

$B = \begin{bmatrix} 2 & 3 \\ 2 & 3 \end{bmatrix}$

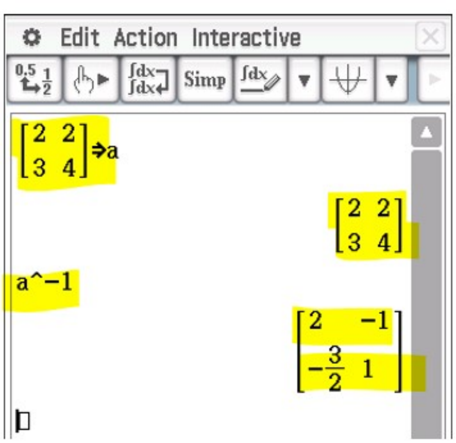
$$\det(B) = 2 \times 3 - 2 \times 3$$
$$= 6 - 6$$
$$= \underline{0}$$

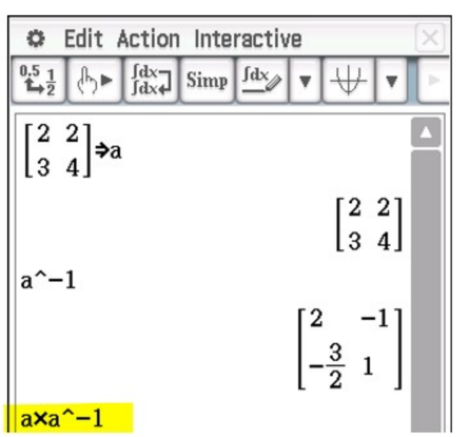
$C = \begin{bmatrix} 2 & 4 \\ 2 & 3 \end{bmatrix}$

$$\det(C) = 2 \times 3 - 2 \times 4$$
$$= 6 - 8$$
$$= \underline{-2}$$

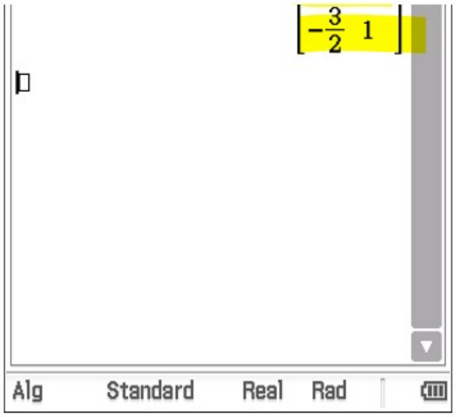
How does this help us divide matrices?

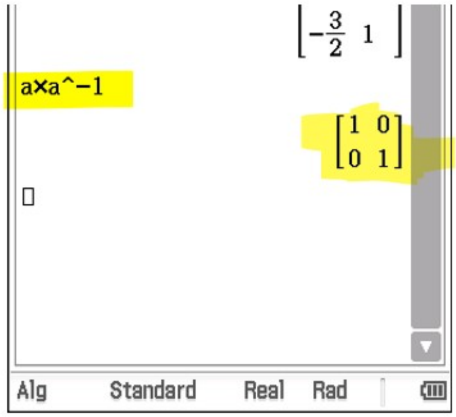
Well, something very interesting happens when you multiply a matrix by it's inverse.
I can spend hours showing how to do this by hand … or just use the CAS!

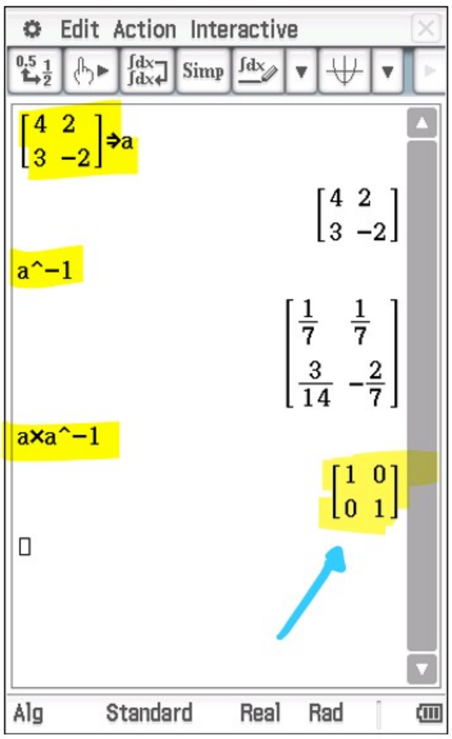


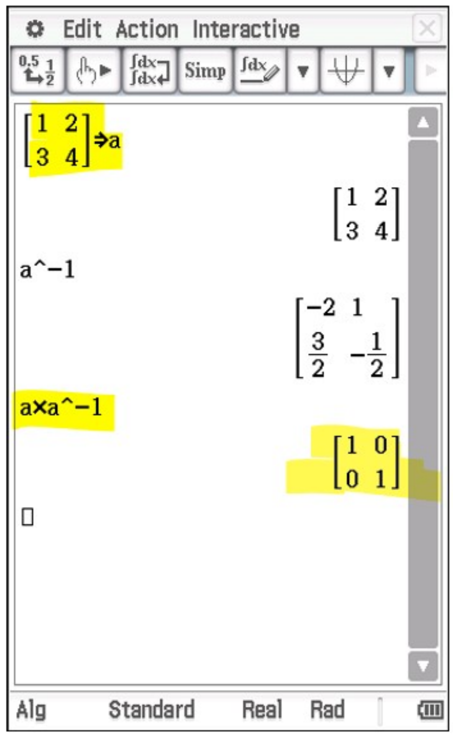


$$a \times a^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$









$a \times a^{-1} = I$

$a^{-1} \times a = I$

So, multiplying a matrix by it's inverse gives me the identity matrix!

How do we use this to help us divide?

That's for the next video!!!!