Smoothing a time series using moving means

Thursday, 23 January 2020 8:17 pm

★ By the end of the lesson I would hope that you have an understanding and be able to apply to questions the following concepts:

- Understand what it means to smooth a time series
- Understand what a moving mean is.
- Understand that you can have a three-moving mean and five-moving mean
- Understand why we would want to use moving means to smooth data.

RECAP:

In the previous lesson we looked at the ideas what time-series can have certain patterns:

- Trend
- Cycle
- Seasonality
- Structural change
- Outliers
- Irregular random fluctuations

If we can see the patterns we can describe them. Not all patterns are obvious as a result of the random fluctuations. So, we use a method called smoothing to take away much of the nature of the fluctuations.

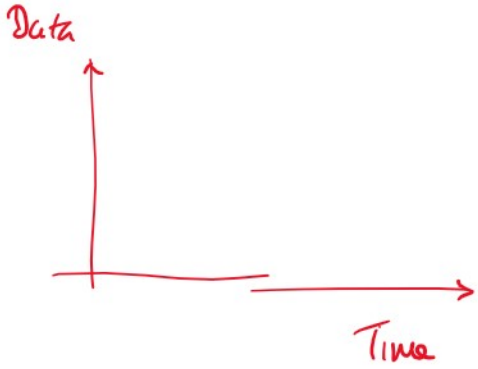
RECAP: Finding the mean of some data

To find the average (or mean) of some data, you add all the data items together and divide by the number of data items.

For time series we look at smoothing the y-values:

$$Mean = \frac{y_1 + y_2 + y_3}{3}$$

This would give a three-moving mean point



Example

The following example has been taken (with permission) from the Cambridge Further Mathematics Units 3 and 4 Textbook

The table below gives the temperature (°C) recorded at a weather station at 9.00 a.m. each day for a week.

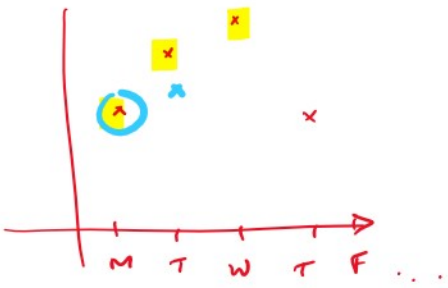
Day	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Temperature	18.1	24.8	26.4	13.9	12.7	14.2	24.9

- Calculate the three-mean smoothed temperature for Tuesday.
- Calculate the five-mean smoothed temperature for Thursday.

3 mean $\quad \frac{18.1 + 24.8 + 26.4}{3} = 23.1$

5 mean : $\frac{24.8 + 26.4 + 13.9 + 12.7 + 14.2}{5}$

$= 18.4$



Example

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Example

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The following table gives the number of births per month over a calendar year in a country hospital. Use the three-moving mean and the five-moving mean methods, correct to one decimal place, to complete the table.

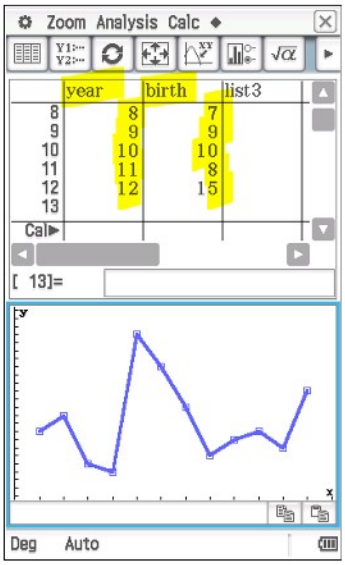
list 1 list 2

	Month	Number of births	3-moving mean	5-moving mean
1	January	10		
2	February	12	9.3	
3	March	6	7.7	11.0
4	April	5	11.0	12.6
5	May	22	15.0	12.8
6	June	18	17.7	13.0
7	July	13	12.7	13.8
8	August	7	9.7	11.4
9	September	9	8.7	9.4
10	October	10	9.0	9.8
11	November	8	11.0	
12	December	15		

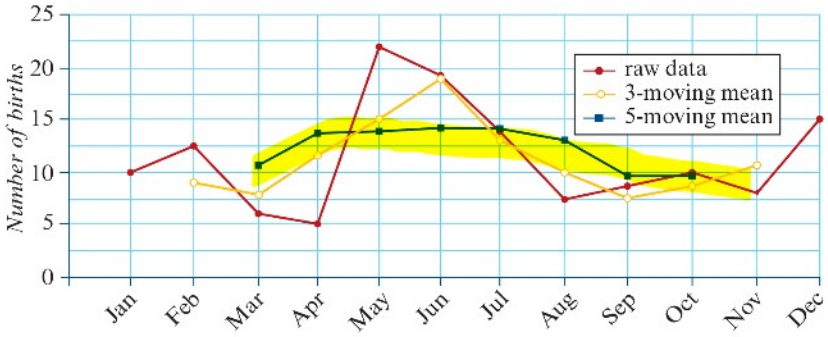
list 3	3 um
2	9.3
3	7.7
4	11.0
5	
6	
7	
8	
9	
10	
11	

list 5	
3	11.0
4	12.6
5	12.8
6	
7	
8	
9	
10	

Plotting the raw data on the CAS gives:



Trying to plot the other data causes a problem due to an "Undefined" problem!



Having plotted the data we need to **interpret it**.

We can see:

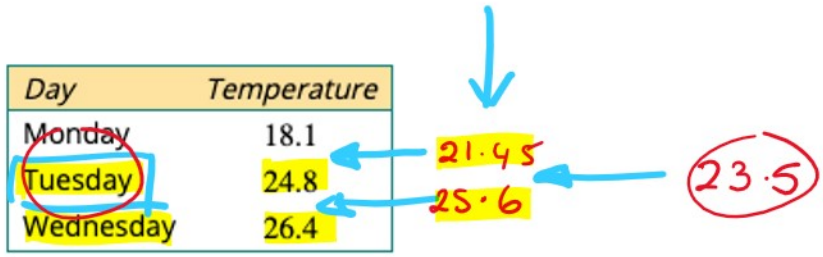
- The five-mean smoothing seems to be better at removing the random fluctuations than the three-mean smoothing.
- The five-mean smoothing seems to be suggest that there is no clear trend (although the raw data suggests there might be an increasing trend).

Two-mean smoothing with centring

When we use an odd number of data items to smooth, we have somewhere to place the data item. We use the middle "time" value of each group to plot the value on the graph.

When there is an even number of data items we are trying to smooth we use a process of two-mean smoothing with centring.

We effectively smooth and then smooth again.



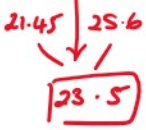
Example

The following example has been taken (with permission) from the Cambridge Further Mathematics Units 3 and 4 Textbook

The temperatures (°C) recorded at a weather station at 9 a.m. each day for a week are displayed in the table.

Day	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Temperature	18.1	24.8	26.4	13.9	12.7	14.2	24.9

Calculate the two-mean smoothed temperature for Tuesday with centring.



Example

The following example has been taken (with permission) from the Cambridge Further Mathematics Units 3 and 4 Textbook

The temperatures (°C) recorded at a weather station at 9 a.m. each day for a week are displayed in the table.

Day	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Temperature	18.1	24.8	26.4	13.9	12.7	14.2	24.9

Calculate the four-mean smoothed temperature for Thursday with centring.

19.45 16.8

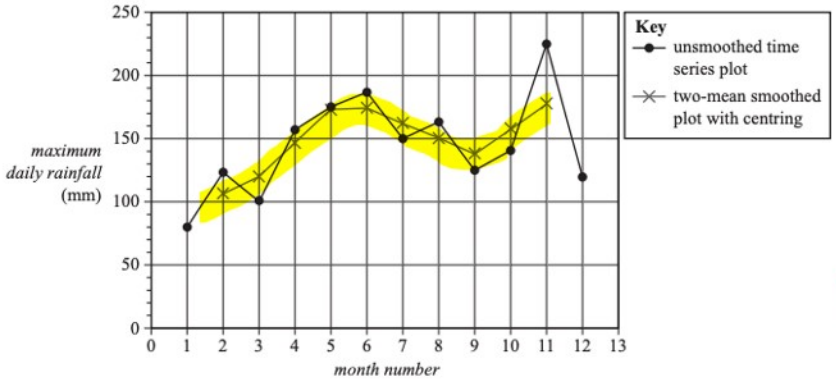
18.1

VCAA Exam Question on this concept
2016 Paper 2

The maximum daily rainfall each month was also recorded at the weather station.
The table below shows the *maximum daily rainfall* each month for a period of one year.

Month	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sep.	Oct.	Nov.	Dec.
Month number	1	2	3	4	5	6	7	8	9	10	11	12
***Maximum daily rainfall* (mm)**	79	123	100	156	174	186	149	162	124	140	225	119

The data in the table has been used to plot *maximum daily rainfall* against *month number* in the time series plot below.



$\frac{124 + 140}{2} = 132$

$\frac{140 + 225}{2} = 182.5$

b. Two-mean smoothing with centring has been used to smooth the time series plot above.
The smoothed values are marked with crosses (×).

Using the data given in the table, show that the two-mean smoothed rainfall centred on October is 157.25 mm. 2 marks

$\frac{182.5 + 132}{2} = 157.25$

VCAA Exam Question on this concept
2017 Paper 1

Question 15
The table below shows the daily *maximum wind speed*, in kilometres per hour, for the days in week 2.

Day	8	9	10	11	12	13	14
***Maximum wind speed* (km/h)**	22	22	19	22	43	37	33

A four-point moving mean with centring is used to smooth the time series data above.
The smoothed *maximum wind speed*, in kilometres per hour, for day 11 is closest to

A. 22
B. 24
C. 26
D. 28
E. 30

26.5 + 30.25

VCAA Exam Question on this concept
2018 Paper 1

Question 15
The table below shows the monthly profit, in dollars, of a new coffee shop for the first nine months of 2018.

Month	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.
Profit ($)	2890	1978	2402	2456	4651	3456	2823	2678	2345

Using four-mean smoothing with centring, the smoothed profit for May is closest to

A. $2502
B. $3294
C. $3503

Using four-mean smoothing with centring, the smoothed profit for May is closest to

A. $2502
B. $3294
C. $3503
D. $3804
E. $4651

$$\frac{3241.25 + 3346.5}{2}$$

$$3293.9$$