Seasonal indices

Thursday, 23 January 2020 8:18 pm

★ By the end of the lesson I would hope that you have an understanding and be able to apply to questions the following concepts:

- Understand what a seasonal index is.
- Understand how to calculate seasonal indices from raw data
- Understand how to deseasonalise data to turn it back to raw data.
- Understand how to interpret the seasonal indices

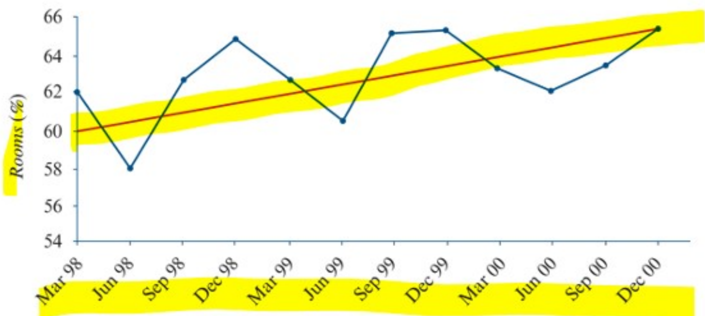
RECAP:

In the previous lessons we have been looking at how to smooth time-series data.

We have looked at how to use:
- three- and five-mean smoothing
- Two- and four-mean smoothing with centering
- Three- and five-median smoothing

We now move onto the idea that time-series data can be seasonal in nature.

We met this in a previous lesson:



Remember: Seasonality is present when there is a periodic movement in a time series that has a calendar-related period – for example a year, a month or a week.

In the summer months we would expect to sell more ice-creams than the winter months.

What if we had a way to comparing the sales of each month with the average of all sales for a seasonal period?

Well … we can!

Seasonal Indices

Seasonal indices tell us how a **particular** season (generally a day, month or quarter) compares to the **average** season.

This means, firstly, we need to find the average sales figure for a whole season.
We then use a formula to help us compare a particular season with the average of the whole season.

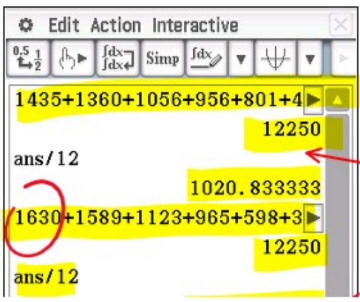
For example.

If we look at the sales of ice creams over a whole year:

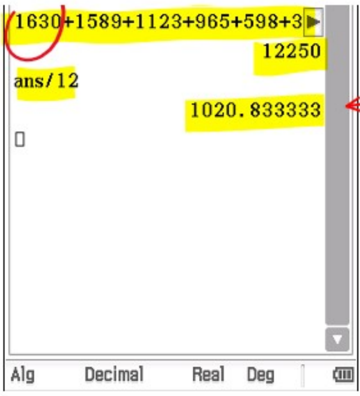
Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Sales	1435	1360	1056	956	801	421	316	598	965	1123	1589	1630

Working out the average we see that, for the whole season we have an average of:

$$\frac{12\ 250}{12} = 1021$$



Why do I do the calculation twice?



Why do I do the calculation twice?

So, the average number of ice-cream sales is: **1021 per month**.

We can now use this to **compare** the actual sales using percentages (or a percentage multiplier).

RECAP: Percentage Multipliers

Percentages can be more than 100!

We can have 110%, 200%, 1000%.
These are normally just a number which helps us relate back to an amount we started with.

Example:
I open a bank account with $50.
At the end of the year I have $60.
I have more than I started with.

Dec → %
× 100

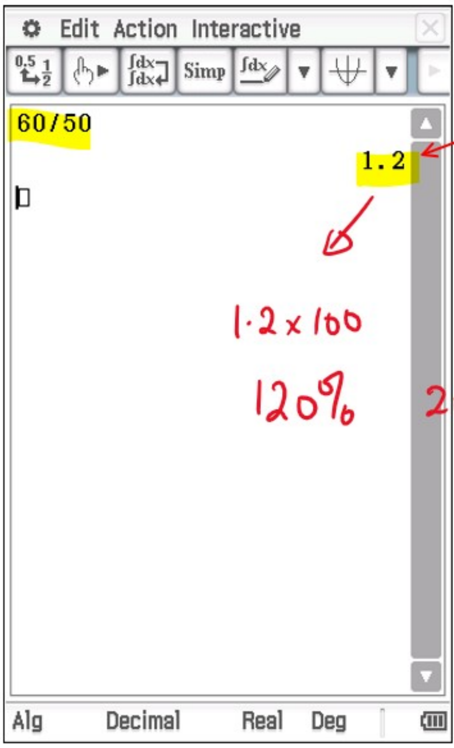
1.2
120% 20% inc

100% → 80% ↓20% dec
0.8

% → Dec
÷ 100

To find out my percentage I do the following:

Percentage increase (or decrease): $\frac{\textit{Amount at the end}}{\textit{Amount at the start}}$



1.2 × 100
120% 20% inc

This is a multiplier.

We can turn it into a percentage by multiplying it by 100.

However, for seasonal indices we leave this figure as a decimal.

We just need to learn to read it that we now have:

120% of what we started with.

Or we have 20% more.

Seasonal indices are just percentage multipliers

So, we can now return to our ice-cream example

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Sales	1435	1360	1056	956	801	421	316	598	965	1123	1589	1630

1021

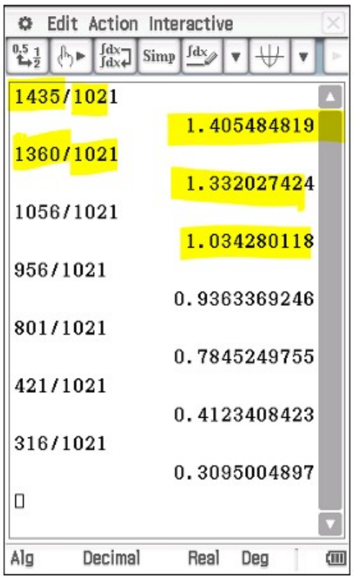
The average per month was 1021.

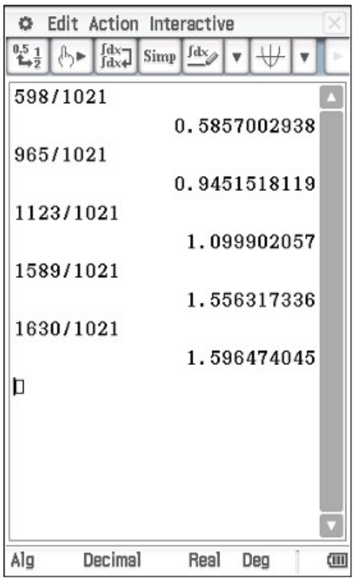
I can now change the sales figures into seasonal indices

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Sales	1435	1360	1056	956	801	421	316	598	965	1123	1589	1630
Index												

The formula is slightly different:

$$\textit{Seasonal Index} = \frac{\textit{Actual figure}}{\textit{Average of Season}} = \frac{\quad}{1021}$$

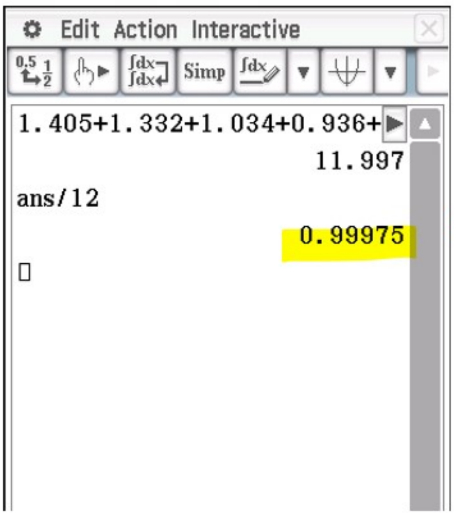




Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Sales	1435	1360	1056	956	801	421	316	598	965	1123	1589	1630
Index	1.405	1.332	1.034	0.936	0.785	0.412	0.310	0.586	0.945	1.100	1.556	1.596

≈ 12

What we should notice, when we take the average of all the indices together, it they should always be 1.

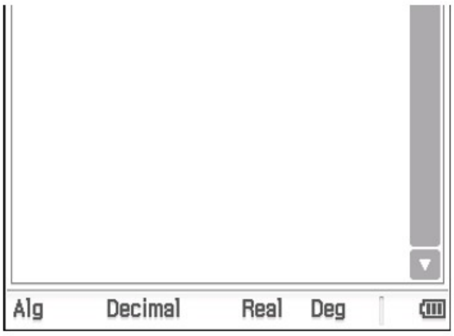


Mine didn't add to one due to rounding errors from the previous calculations. It's going to round to 1 though!

1.405×100

140.5% ↑ 40.5%

$0.785 \times 100 = 78.5\%$



$0 \cdot 785 \times 100 = 78 \cdot 5\%$

$100\% \rightarrow 78 \cdot 5\% = \boxed{21 \cdot 5\%}$

Understanding Seasonal Indices

Now we have the seasonal indices, how do we interpret them?

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Sales	1435	1360	1056	956	801	421	316	598	965	1123	1589	1630
Index	1.405	1.332	1.034	0.936	0.785	0.412	0.310	0.586	0.945	1.100	1.556	1.596

Remember: The average was 1021.

Hence, for January the seasonal index was 1.4 or 140%.
This means we sold **40**% more than the average.

$1 \cdot 405 \times 100 = 140 \cdot 5\%$

In November the seasonal index was 1.6 or 160%.
This means we sold **60**% more than the average.

$100 \rightarrow 140 \cdot 5 \quad + 40 \cdot 5\%$

inc $40 \cdot 5\%$

We need to be careful with figures that are less than one!

In May we had a seasonal index of 0.8 or 80%.
This means we sold only 80% of the monthly average.
Or we sold 20% less than the monthly average.

$1 \cdot 556 \times 100 = 155 \cdot 6\%$ inc $55 \cdot 6\%$

$0 \cdot 412 \times 100$

$41 \cdot 2\%$

$100 \rightarrow 41 \cdot 2\% = -58 \cdot 8\%$

dec $\boxed{58 \cdot 8\%}$

Turning the data from Seasonalised to Deseasonalised and back again

When we deseasonalise data we are looking to remove the variations the seasons add to the data.
This will smooth the data and allow us to look at trends.

(Raw)

Example

The following example has been taken (with permission) from the Cambridge Further Mathematics Units 3 and 4 Textbook

The seasonal index (SI) for cold drink sales for summer is SI=1.33.
Last summer a beach kiosk's actual cold drink sales totalled $15653.
What were the deseasonalised sales?

Raw

We need a new formula for this!

SI = 1·33

Act : 15 653

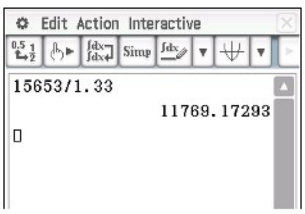
$$deseasonalised\ figure = \frac{actual\ figure}{seasonal\ index}$$

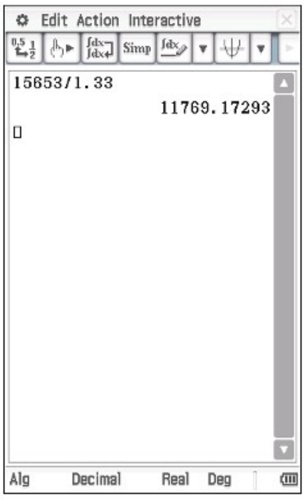
Hence, we know the actual figure is: $15 653
The seasonal index was 1.33

Hence, the deseasonalised figure is $11 769

$$des = \frac{15\ 653}{1 \cdot 33}$$

$$= \underline{\underline{11\ 769 \cdot 17}}$$





= 11 769.17

To turn the data back into a **seasonal value** we can reverse the formula

$$actual\ figure = (deseasonalised\ figure) \times seasonal\ index$$

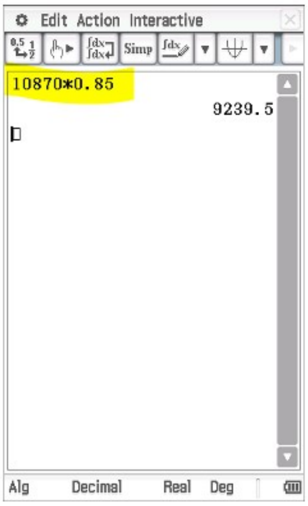
Example
The following example has been taken (with permission) from the Cambridge Further Mathematics Units 3 and 4 Textbook

The seasonal index for cold drink sales for spring is SI=0.85.
Last spring a beach kiosk's deseasonalised cold drink sales totalled $10870.
What were the **actual** sales?

SI : 0.85

des : 10870

Using the formula above we find that the actual sales figure is $9 239.50



Act = des × SI

= 10870 × 0.85

= 9239.5

Deseasonalising a time-series

We generally deseasonalise data which covers a number of seasons (and years).

Example
The following example has been taken (with permission) from the Cambridge Further Mathematics Units 3 and 4 Textbook

The quarterly sales figures for Mikki's shop over a 3-year period are given.

Year	*Summer*	*Autumn*	*Winter*	*Spring*
1	920	1085	1241	446
2	1035	1180	1356	541
3	1299	1324	1450	659

Use the seasonal indices shown to deseasonalise these sales figures. Write answers correct to the nearest whole number.

Use the seasonal indices shown to deseasonalise these sales figures. Write answers correct to the nearest whole number.

Summer	Autumn	Winter	Spring
1.03	1.15	1.30	0.52

$$\frac{1085}{1.15}$$

Remember: To deseasonalise we use the following formula

$$deseasonalised\ figure = \frac{actual\ figure}{seasonal\ index}$$

Year	Summer	Autumn	Winter	Spring
1	893	943		
2	1005			
3	1261			

Plotting the deseasonalised data

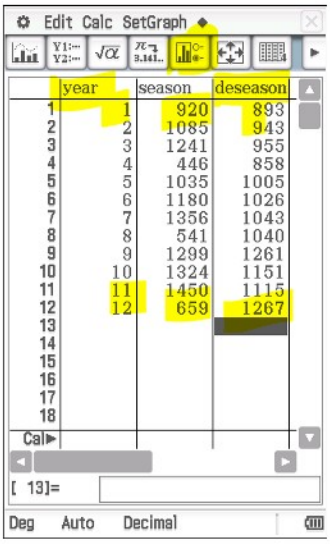
Using the data from above.

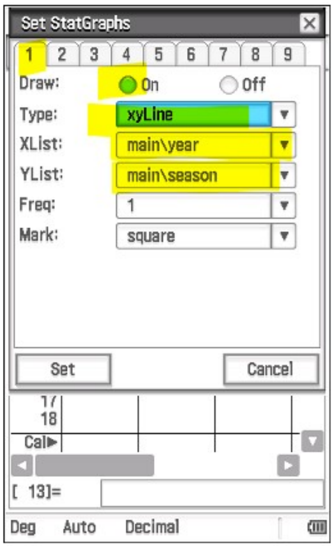
Year	Summer	Autumn	Winter	Spring
1	893	943	955	858
2	1005	1026	1043	1040
3	1261	1151	1115	1267

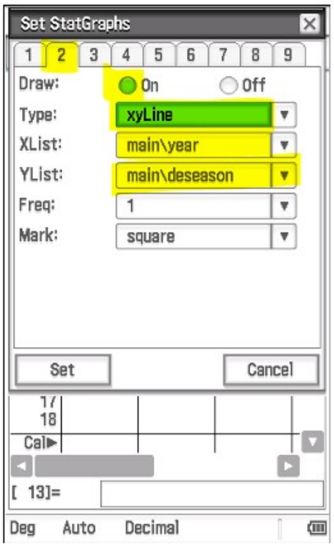
1	S	1
	A	2
	W	3
	S	4

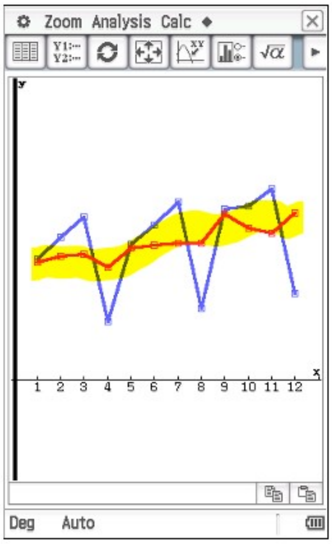
3	1261	1151	1115	1267

W 3
S 4
2 S 5
A 6
W .
S .
3 S .
A .
W .
S 12









How did they get the seasonal indices from multiple year's worth of data?

We have only worked out seasonal indices for one year's worth of data.
We can work seasonal indices for several years worth of data.

Let's look at how they have gone from:

Year	*Summer*	*Autumn*	*Winter*	*Spring*
1	920	1085	1241	446
2	1035	1180	1356	541
3	1299	1324	1450	659

To ...

Summer	*Autumn*	*Winter*	*Spring*
1.03	1.15	1.30	0.52

Note: 1.03 for Summer is and **average of all the summer seasonal indices.**

Firstly, we need to work out the seasonal indices for each season in each year.

Year 1:

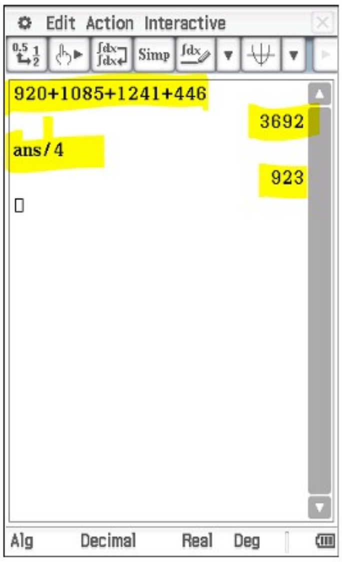
Year	Summer	Autumn	Winter	Spring
1	920	1085	1241	446

Year 1:

Year	Summer	Autumn	Winter	Spring
1	920	1085	1241	446

Work out the **year average.**

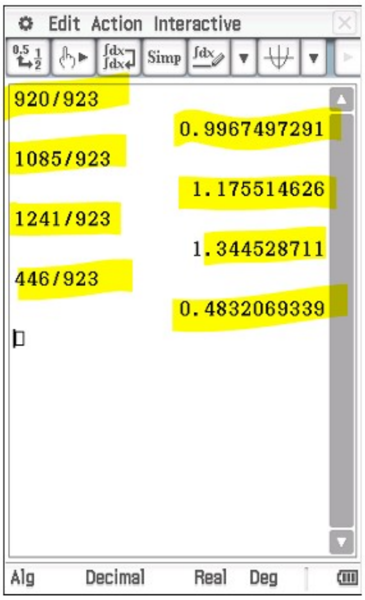
$$SI = \frac{Act}{Aver}$$



$$\frac{920}{923} \quad \frac{1085}{923} \quad \frac{1241}{923}$$

Use this to find the seasonal indices for Year 1

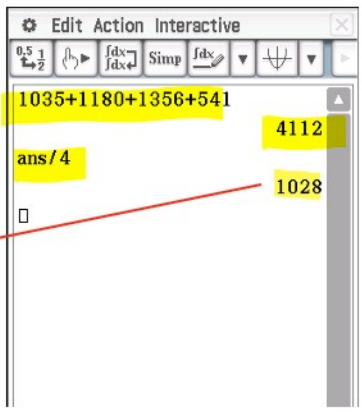
Year	Summer	Autumn	Winter	Spring
1	920	1085	1241	446
Index	0.997	1.176	1.345	0.483

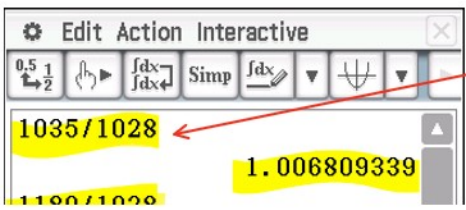


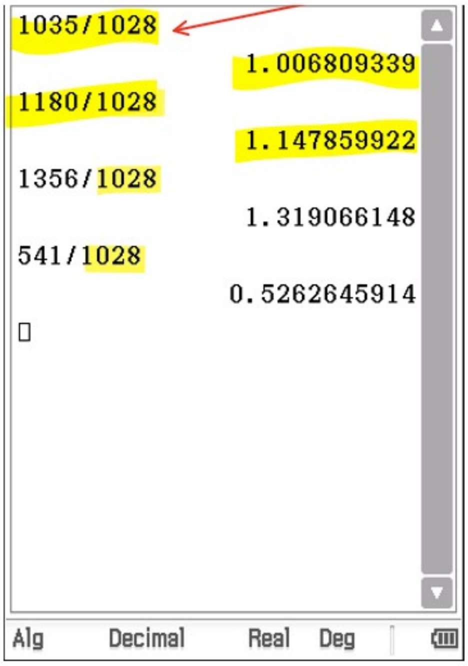
Repeat the process for Year 2:

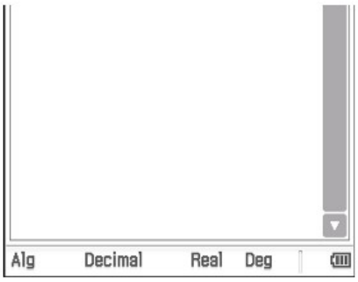
Year	Summer	Autumn	Winter	Spring
1	1035	1180	1356	541
Index	1.007	1.148	1.319	0.526

REMEMBER:
YOU MUST WORK OUT THE NEW AVERAGE









Repeat for all other years!

Putting the seasonal indices together in one table we see the following:

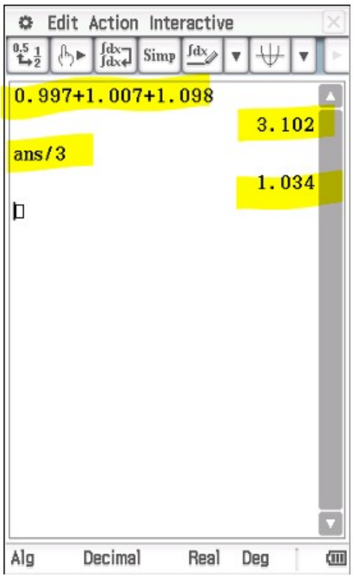
Year	Summer	Autumn	Winter	Spring
1	0.997	1.176	1.345	0.483
2	1.007	1.148	1.319	0.526
3	1.098	1.119	1.226	0.557

We can now calculate the 3-year averaged seasonal indices by taking the averages of each season.

Summer	Autumn	Winter	Spring
1.034	1.148	1.300	0.52

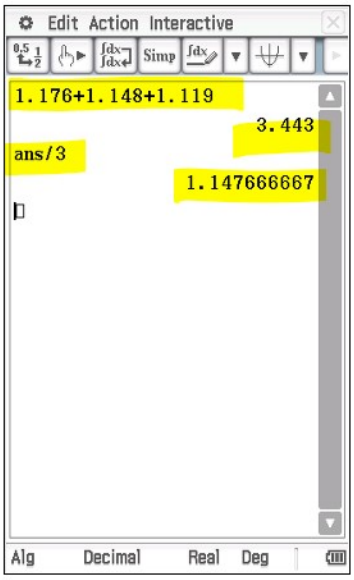
Average of Summer

Average the red numbers.



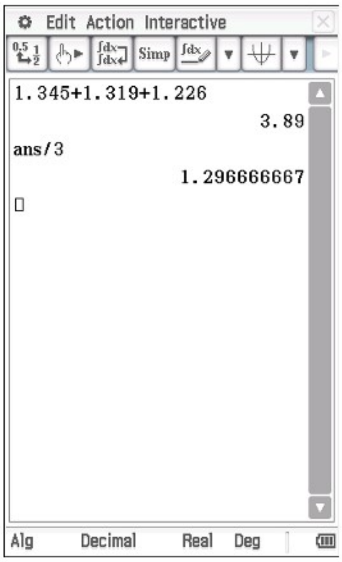
Average of Autumn

Average the blue numbers.



Average of Winter

Average the green numbers.



Note about correcting for seasonality

When we use the formula shown below we need to understand what it means

$$\text{deseasonalised figure} = \frac{\text{actual figure}}{\text{seasonal index}}$$

$$\text{des} = \frac{\text{act}}{1} \times \frac{1}{\text{SI}}$$

This is the same as saying

$$\text{deseasonalised figure} = \text{actual figure} \times \frac{1}{\text{seasonal index}}$$

So, if a seasonal index was 1.30 (for example) then

$$\text{deseasonalised figure} = \text{actual figure} \times \frac{1}{1.3}$$

$$\text{des} = \text{act} \times \frac{1}{1.3}$$

$$\text{deseasonalised figure} = \text{actual figure} \times 0.769$$

100%

act × 77%

This means the deseasonalised figure is approximately 23% less than the seasonal figure.

↓ 23%

S.I = 0.7

$$\text{des} = \text{act} \times \frac{1}{0.7}$$

$$= \text{act} \times 1.429$$

100% → 143%

↑ 43%

$= act \times 1.429$

$= act \times 143\%$ $\uparrow 43\%$