Pythagoras’ theorem

Sunday, 20 October 2019 11:13 am

By the end of the lesson I would hope that you have an understanding (and be able to apply to questions) the following concepts:

- Know what Pythagoras' Theorem is
- Know that it applies to right angled triangles
- Know that we can use it to find the length of unknown sides
- Know what Pythagorean Triads are

RECAP

In previous lessons we have looked at reviewing the work on parallel lines, triangles and the properties of polygons. It's now time to start to use some of the theory by recapping the work we started in Year 9 on Pythagoras' Theorem.

RECAP: Pythagoras' theorem using a diagram

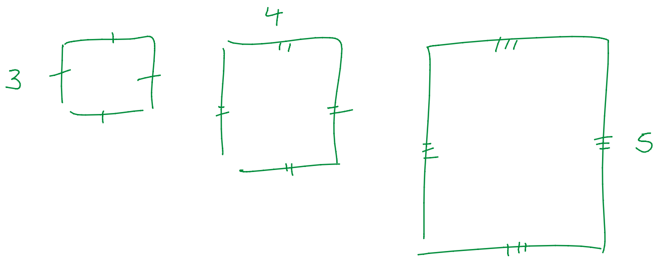
We all know Pythagoras' theorem.

$$c^2 = a^2 + b^2$$

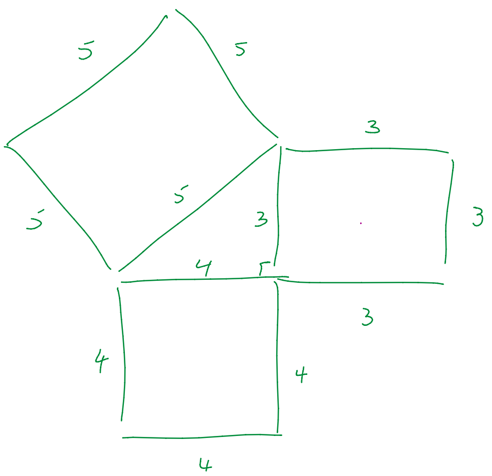
$a^2 = b^2 + c^2$

But where did it come from?
Why does it work?

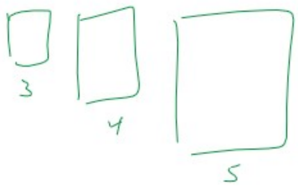
Look at the following three squares.



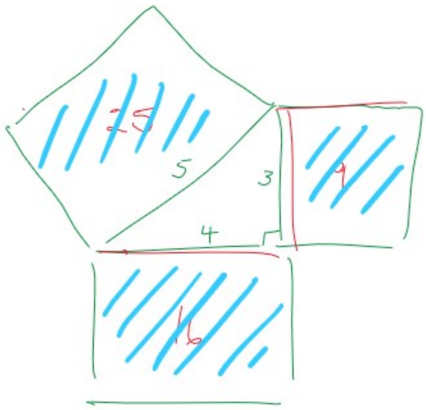
We know, that for certain squares, if we join them together in a certain way, they form a right angles triangle. This might be how they built the pyramids.

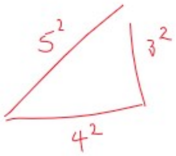


When we look at squares ... we normally like to find their area (or perimeter) as it's the easiest area of find. So, let's look at finding the area of each square ...



$9+16=25$



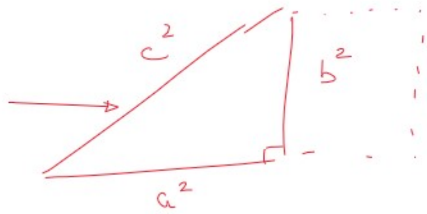


$3^2+4^2=5^2$

Now ... what do you notice about the areas of the two smaller squares when we add them together?
Yup! The areas of the two smaller squares, when added together, equals the area of the bigger square.
And so ... you've just proven the formula.

$$c^2 = a^2 + b^2$$

Hypotenuse!

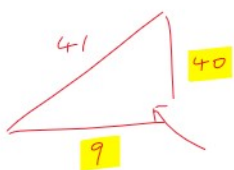


Pythagorean Triads

Do all squares meet to make right angled triangles?
NOPE.
But there are quite a lot.
The more common ones are called Pythagorean Triads

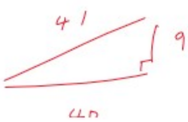
Here is a nice table with the triads (thanks Cambridge!)

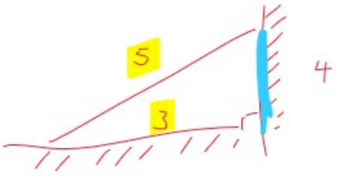
a	3	5	7	8	9	11
b	4	12	24	15	40	60
c	5	13	25	17	41	61

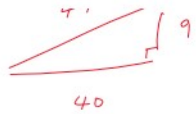


$3+4 = 4+3$

There are certain triangles which exam questions use A LOT!
3-4-5 and 5-12-13 triangles are used a LOT!



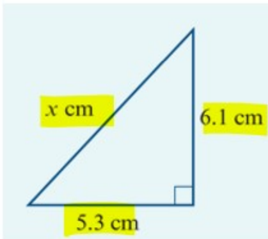




Examples

The following examples are used, with permission, from the Cambridge Further Mathematics Units 3 and 4 textbook.

Example 1
Find the value, correct to two decimal places, of the unknown length for the triangle opposite.



$c^2 = a^2 + b^2$

$x^2 = 5.3^2 + 6.1^2$

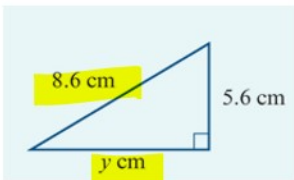
$x^2 = 28.09 + 37.21$

$x^2 = 65.3$

$x = \sqrt{65.3}$

$x = \underline{8.08}$ cm

Example 2
Find the value, correct to two decimal places, of the unknown length for the triangle opposite.



$c^2 = a^2 + b^2$

$8.6^2 = y^2 + 5.6^2$

$8.6^2 - 5.6^2 = y^2$

$y^2 = 42.6$

$y = \sqrt{42.6}$

$y = \underline{6.53 \text{ cm}}$

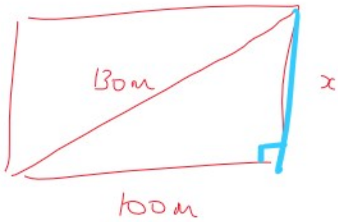
Example 3

The diagonal of a soccer ground is 130 m and the long side of the ground measures 100 m. Find the length of the short side, correct to the nearest cm.

$$c^2 = a^2 + b^2$$

$$130^2 = 100^2 + x^2$$

$$x = 83.06623863 \text{ m}$$



$$x = 8306.623863 \text{ cm}$$

$$= \underline{\underline{8307 \text{ cm}}}$$

$$x = \underline{\underline{83.07 \text{ m}}}$$