Properties of regular polygons: a review

Sunday, 20 October 2019 10:57 am

By the end of the lesson I would hope that you have an understanding (and be able to apply to questions) the following concepts:

- Know what a polygon is
- Know what a regular polygon is
- Know what a convex polygon is
- Know what it means by the sum of the interior angles
- Know how to find the sum of the interior angles of a regular polygon
- Know where the angle bisectors meet

RECAP

This is a continuation of the section which is reviewing all the work which has taken place in previous year levels of Mathematics. We have looked at the properties of parallel lines and triangles. Now it's time to move it up a level to look at the properties of regular polygons.

What is a POLYGON?



A missing parrot!!!!

The jokes don't get much better!

polygon

/ˈpɒlɪg(ə)n/

noun GEOMETRY

noun: **polygon**; plural noun: **polygons**

a plane figure with at least three straight sides and angles, and typically five or more.

Origin

GREEK polugōnos → GREEK polugōnon → LATE LATIN polygon

late 16th century

Hold on … what's a plane?

plane[1]
/pleɪn/

See definitions in: Mathematics | Transportation | Physics | Tools

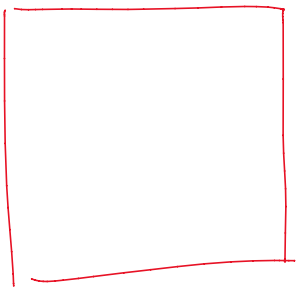
noun
noun: **plane**; plural noun: **planes**

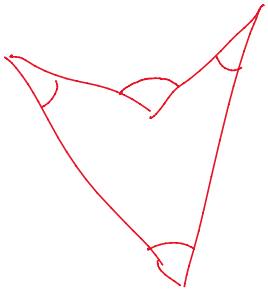
1. a flat surface on which a straight line joining any two points on it would wholly lie.
   "the horizontal plane"
   Similar: flat surface | level surface | the flat | horizontal
   - an imaginary flat surface through or joining material objects.
     "the planets orbit the sun in roughly the same plane"
   - a flat or level surface of a material object.
     "the **plane of** his forehead"
   - a flat surface producing lift by the action of air or water over and under it.
2. a level of existence, thought, or development.
   "everything is connected on the spiritual plane"
   Similar: level | stage | degree | standard | stratum | position | rung

OK!
So a polygon is a shape with three or more sides and angles.

An example of a polygon:

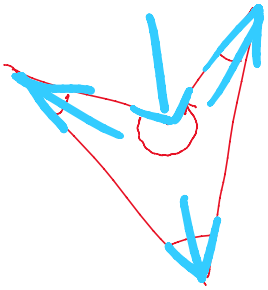
Here is an example of a polygon with 4 sides.
The sides are all straight.
What do you notice about one of the angles though?

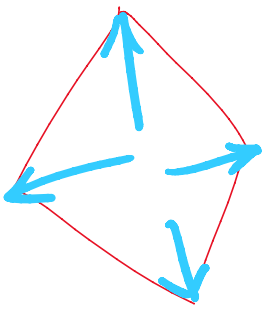




A convex polygon

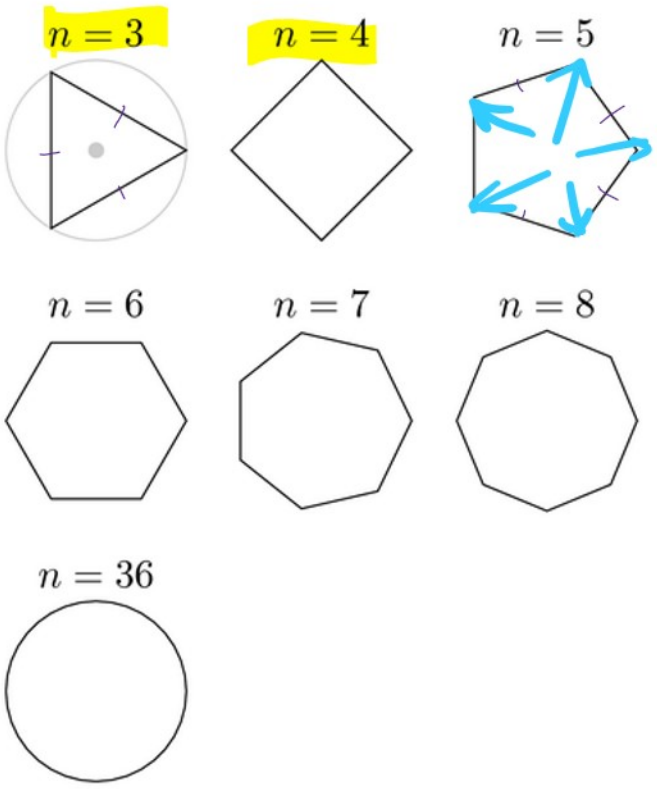
A convex polygon has all angles less than 180°.
I like to think of drawing arrows and seeing if the arrows are pushing out or in.
Here is an example of a convex polygon.





A regular polygon

A regular polygon is a polygon where all the sides (and internal angles) are the same size.
Here are some examples of regular polygons:



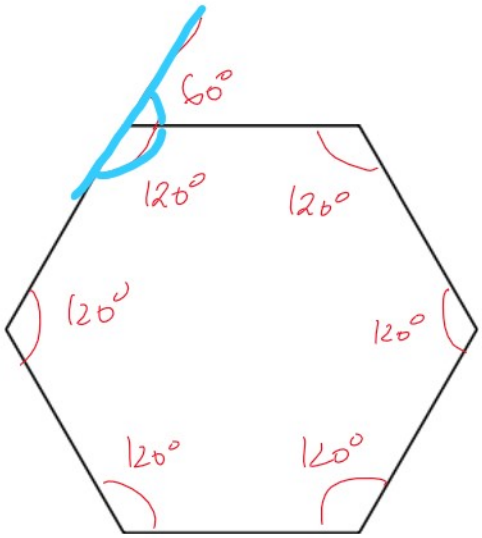
The n stands for the number of sides the polygon has.
Each of the above shapes has a name describing the polygon.



Interior and exterior angles

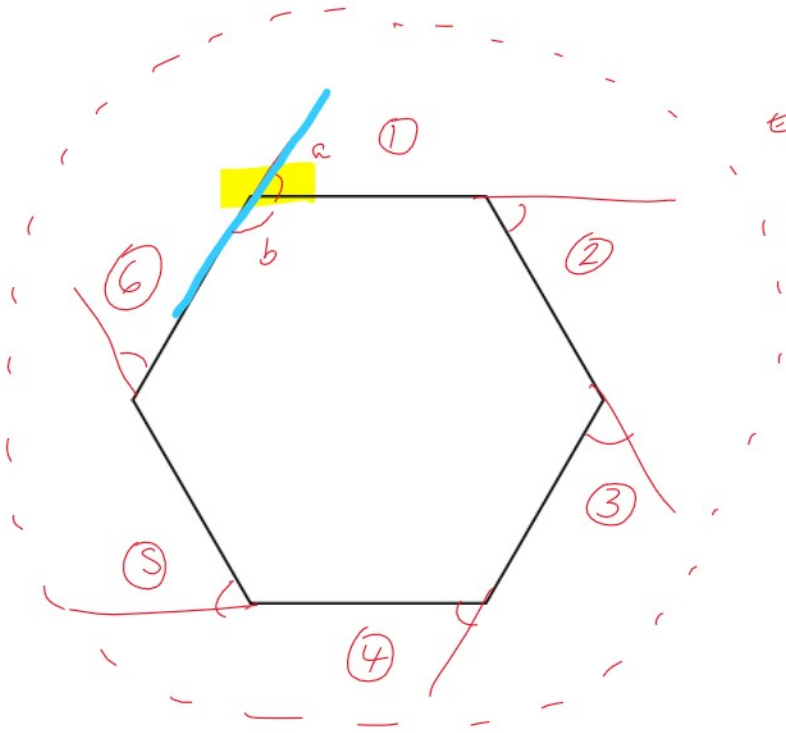
Like in the previous video, polygons have interior and exterior angles.
Interior angles are inside.

Int = 180° − Ext
= 180° − 60
= 120°



Sum = 6 × 120°
= 720°

Exterior angles are outside:



$b° = 180° - a°$

Ext = 360°
6
= 60°

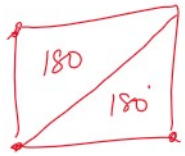
Finding the formula for the sum of the interior angles

We can use a formula to find the sum of interior angles
The formula is:

$$Sum = 180 \times (n - 2)$$

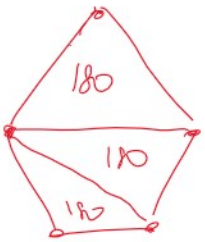
$$Sum = 180n - 360$$

4



S = 180 × 2

5



S = 180 × 3
= 540°

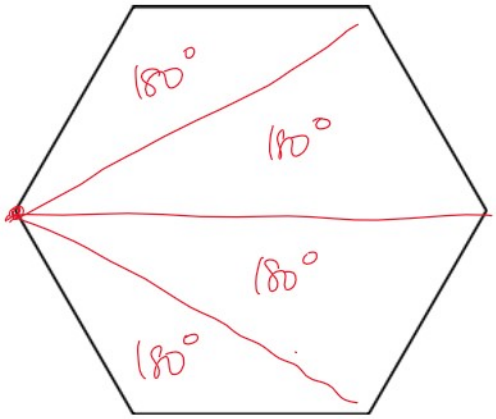
This is why ...
Split the polygon into equal triangles.
We know the triangles each have 180° inside.

6

$S = 180 \times 4$

$= 720°$

$S = 180 \times (n-2)$

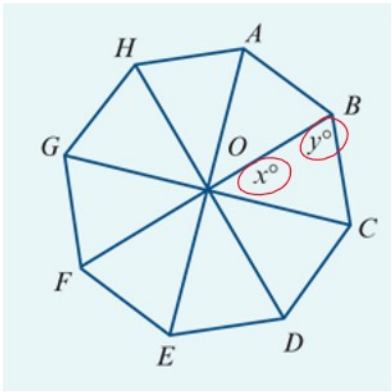


Examples

The following examples are used, with permission, from the Cambridge Further Mathematics Units 3 and 4 textbook.

Example 1

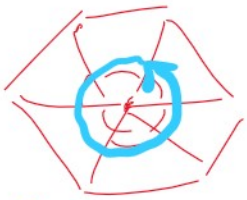
The diagram opposite shows a regular octagon.



Show that $x = 45$.

Find the size of angle y.

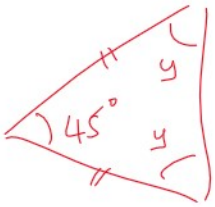
6



$x = \frac{360°}{8}$

$= 45°$

360°



$180° - 45° = 2y$

$2y = 135°$

$y = 67.5°$

Example 2
Find the sum of the interior angles of an eight-sided convex polygon (octagon).

$$\text{Sum} = 180 \times (n-2)$$
$$= 180 \times (8-2)$$
$$= 180 \times 6$$
$$= 1080^\circ$$