Problems in three dimension

Sunday, 19 April 2020 10:24 am

★ By the end of the lesson I would hope that you have an understanding (and be able to apply to questions) the following concepts:

- Understand what angles of elevation and depression are
- Understand what bearings are
- Understand the concept of triangulation and how to apply it to questions

RECAP

This is the final lesson in this series. It's going to build on all the knowledge gained in the previous videos but using three dimensions.

It's not all about two dimensions

Whilst we spend a lot of time in Mathematics doing things on a flat plane, we must realise that there are lots of applications where we need to look at the third dimension.

Whilst we are adding a third dimension, we must remember that a triangle is a triangle regardless of how many dimensions it has!

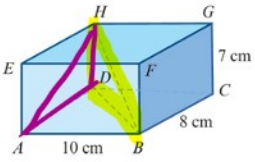
Let's look at some examples of how to apply the Mathematics in three dimensions

Examples

The following examples have been extracted, with permission, from the Cambridge Further Mathematics Units 3 and 4 textbook.

$ABCDEFGH$ is a cuboid. Find:

- distance DB
- distance HB
- the magnitude of angle HBD
- distance HA
- the magnitude of angle HBA.



a) $DB^2 = 8^2 + 10^2$

$= 64 + 100$

$= 164$

$DB = \sqrt{164}$

$= 12.81$ cm

b) $HB^2 = 7^2 + (\sqrt{164})^2$

$= 49 + 164$

$= 213$

$HB = \sqrt{213}$

$= 14.59$ cm

c) $\tan\theta = \frac{7}{\sqrt{164}}$

$\theta = \tan^{-1}\left(\frac{7}{\sqrt{164}}\right)$

$\theta = 28.66°$

d) $HA^2 = 7^2 + 8^2$

$= 49 + 64$

$= 113$

$HA = \sqrt{113}$

$= 10.63$ cm

e) $\cos B = \frac{a^2 + c^2 - b^2}{2ac}$

$\cos\theta = \frac{(\sqrt{213})^2 + (10)^2 - (\sqrt{113})^2}{2 \cdot \sqrt{213} \cdot 10}$

A B

$HA = \sqrt{113}$

$= 10.63$ cm

$\cos$ $= \dfrac{}{2 \cdot \sqrt{213} \cdot 10}$

$= \dfrac{213 + 100 - 113}{20\sqrt{213}}$

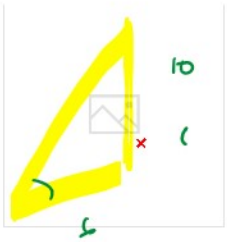
$\cos\theta = \dfrac{200}{20\sqrt{213}}$

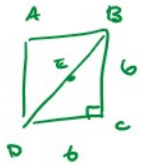
$\theta = \cos^{-1}\left(\dfrac{200}{20\sqrt{213}}\right)$

$\theta = 46.75^\circ$

The diagram shows a pyramid with a square base. The base has sides 6 cm long and the edges VA, VB, VC, VD are each 10 cm long.

- Find the length of DB.
- Find the length of BE.
- Find the length of VE.
- Find the magnitude of angle VBE.
- Give all answers correct to two decimal places.





$DB^2 = 6^2 + 6^2$

$= 36 + 36$

$= 72$

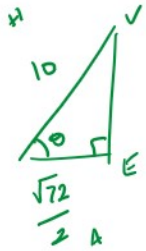
$DB = \sqrt{72}$

$= 8.49$ cm

$BE = \frac{1}{2} DB$

$= \frac{1}{2}\sqrt{72}$

$= 4.24$ cm



$10^2 = \left(\frac{\sqrt{72}}{2}\right)^2 + VE^2$

$100 - \left(\frac{\sqrt{72}}{2}\right)^2 = VE^2$

$VE^2 = 82$

$VE = \sqrt{82}$

$VE = 9.01$ cm

$\cos\theta = \dfrac{\sqrt{72}/2}{10}$ $\theta = \cos^{-1}\left(\dfrac{\frac{\sqrt{72}}{2}}{10}\right)$

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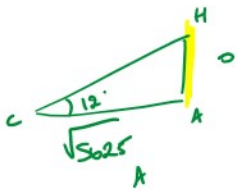
$\theta = 64.90^\circ$

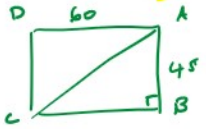
A communications mast is erected at corner A of a rectangular courtyard $ABCD$ whose sides measure 60 m and 45 m. If the angle of elevation of the top of the mast from C is 12°, find:

- the height of the mast
- the angle of elevation of the top of the mast from B (where AB=45 m).

Give answers correct to two decimal places.







$AC^2 = 60^2 + 45^2$

$= 3600 + 2025$

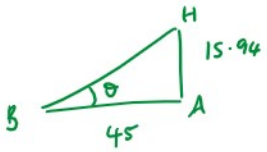
$= 5625$

$AC = \sqrt{5625}$

$\tan 12^\circ = \frac{AH}{\sqrt{5625}}$

$AH = \tan 12^\circ \times \sqrt{5625}$

$= 15.94\text{m}$



$\tan\theta = \frac{15.94}{45}$

$\theta = \tan^{-1}\left(\frac{15.94}{45}\right)$

$= 19.51^\circ$