Matrix arithmetic: addition, subtraction and scalar multiplication

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By the end of the lesson I would hope that you have an understanding and be able to apply to questions the following concepts:

- Understand the conditions which must be met to be able to add and subtract matrices
- How to add two matrices
- How to subtract two matrices
- How to do scalar multiplication
- What the zero matrix is
- What multiplying by the identity matrix does

RECAP

We now understand what a matrix is for an how to apply it's use to real world situations.
But, other than expressing real world things as a matrix, what is the point of matrices!?
All in good time grasshopper.

Let's look at some more maths relating to Matrices first ...

Adding Matrices together

There is a need to be able to add Matrices together.
The key to doing this is to add identical elements together in each Matrix.

Remember what an element is?

$$A = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 2 & 4 & 6 & 8 \\ 4 & 3 & 2 & 1 \\ 9 & 8 & 7 & 6 \end{bmatrix}$$

a_{23}

(4 by 4)

a_{41}

So, can I add the following matrices together?

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \qquad \begin{bmatrix} 1 \\ 2 \\ 3 \\ \boxed{4} \end{bmatrix}$$

!

The answer is therefore a resounding NO!
The matrices must be of the same **order** for us to be able to add and subtract them.

Therefore, we can add the following:

$$\underset{2 \times 2}{\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}} + \underset{2 \times 2}{\begin{bmatrix} 4 & 5 \\ 1 & 3 \end{bmatrix}} = \underset{2 \times 2}{\begin{bmatrix} 5 & 7 \\ 4 & 7 \end{bmatrix}}$$

We can also subtract them!
But we'll end up with negative numbers!

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} - \begin{bmatrix} 4 & 5 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} -3 & -5 \\ 2 & 1 \end{bmatrix}$$

(2×2) (2×2) (2×2)

Example

The following example has been extracted, with permission, from the Cambridge Further Maths Units 3 and 4 Textbook.

$A = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 4 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -2 & 1 \end{bmatrix}$ Find $A + B$.

$$\begin{bmatrix} 2 & 3 & 0 \\ 1 & 4 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 \\ 2 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 5 & 3 \\ 3 & 2 & 3 \end{bmatrix}$$

$4 + -2$
$4 - 2$

Example

The following example has been extracted, with permission, from the Cambridge Further Maths Units 3 and 4 Textbook.

$A = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 4 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -2 & 1 \end{bmatrix}$ Find $A - B$.

$$\begin{bmatrix} 2 & 3 & 0 \\ 1 & 4 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 2 & 3 \\ 2 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & -3 \\ -1 & 6 & 1 \end{bmatrix}$$

$4 - -2$
$4 + 2$

Multiplying Matrices by a scalar

OK! Barry … just stop it!
SCALAR??? What is a SCALAR?

Basically a scalar is a number.

Multiplying a matrix by a scalar is really easy.

You basically multiply every number inside the matrix by the scalar (number)

Example

The following example has been extracted, with permission, from the Cambridge Further Maths Units 3 and 4 Textbook.

Find 3 A and 0.5 C

$A = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 4 & 2 \end{bmatrix}$ $C = \begin{bmatrix} 4 & -4 \\ -2 & 6 \end{bmatrix}$

$P = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$3A = 3 \times \begin{bmatrix} 2 & 3 & 0 \\ 1 & 4 & 2 \end{bmatrix}$

$= \begin{bmatrix} 6 & 9 & 0 \\ 3 & 12 & 6 \end{bmatrix}$

$4P = 4 \times \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$4P = \begin{bmatrix} 4 & 8 \\ 12 & 16 \end{bmatrix}$

$0.5C = \frac{1}{2}C = \frac{1}{2} \times \begin{bmatrix} 4 & -4 \\ -2 & 6 \end{bmatrix}$

$$0.5C = \frac{1}{2}C = \frac{1}{2} \times \begin{bmatrix} 4 & -4 \\ -2 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -2 \\ -1 & 3 \end{bmatrix}$$

The Zero Matrix

Possibly one of the stupidest things I've ever heard of!
The prize goes to Barry for this one!

The zero matrix is a matrix which is filled with only zeros. It must be called **O**
Note: It doesn't have to be square!

$$O = [0] \qquad O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad O = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Example

The following example has been extracted, with permission, from the Cambridge Further Maths Units 3 and 4 Textbook.

Show that $3\boldsymbol{G} - 2\boldsymbol{H} = \boldsymbol{O}$

$$G = \begin{bmatrix} 6 & 0 \\ -4 & 2 \end{bmatrix} \qquad H = \begin{bmatrix} 9 & 0 \\ -6 & 3 \end{bmatrix}$$

$$= 3 \times \begin{bmatrix} 6 & 0 \\ -4 & 2 \end{bmatrix} - 2 \times \begin{bmatrix} 9 & 0 \\ -6 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 18 & 0 \\ -12 & 6 \end{bmatrix} - \begin{bmatrix} 18 & 0 \\ -12 & 6 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

2by2 2by2 2by2

The CAS can do all this for you too!

If $\boldsymbol{A} = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 4 & 2 \end{bmatrix}$ and $\boldsymbol{B} = \begin{bmatrix} 1 & 0 & 3 \\ 2 & -2 & 1 \end{bmatrix}$, find:

- **a** $A + B$
- **b** $A - B$
- **c** $3A - 2B$

a) $\begin{bmatrix} 3 & 3 & 3 \\ 3 & 2 & 3 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 3 & -3 \\ -1 & 6 & 1 \end{bmatrix}$

c) $\begin{bmatrix} 4 & 9 & -6 \\ -1 & 16 & 4 \end{bmatrix}$

How does this relate to the real world examples?

Now we have the ability to add sales data together for example.

Example

The following example has been extracted, with permission, from the Cambridge Further Maths Units 3 and 4 Textbook.

The sales data for two used car dealers, Honest Joe's and Super Deals, are displayed below.

	2014			2015		
Car sales	*Small*	*Medium*	*Large*	*Small*	*Medium*	*Large*
Honest Joe's	24	32	11	26	38	16
Super Deals	32	34	9	35	41	12

Construct two matrices, A and B, to represent the sales data for 2014 and 2015 separately.

$$A = \begin{array}{c c} \begin{array}{ccc} S & M & L \end{array} & \\ \begin{bmatrix} 24 & 32 & 11 \\ 32 & 34 & 9 \end{bmatrix} & \begin{array}{c} H \\ S \end{array} \end{array} \qquad B = \begin{array}{c c} \begin{array}{ccc} S & M & L \end{array} & \\ \begin{bmatrix} 26 & 38 & 16 \\ 35 & 41 & 12 \end{bmatrix} & \begin{array}{c} H \\ S \end{array} \end{array}$$

Construct a new matrix $C=A+B$. What does this matrix represent?

$$C = \begin{array}{c c} \begin{array}{ccc} S & M & L \end{array} & \\ \begin{bmatrix} 50 & 70 & 27 \\ 67 & 75 & 21 \end{bmatrix} & \begin{array}{c} H \\ S \end{array} \end{array}$$

Construct a new matrix, $D=B-A$. What does this matrix represent?

$$D = B - A = \begin{array}{c c} \begin{array}{ccc} S & M & L \end{array} & \\ \begin{bmatrix} 2 & 6 & 5 \\ 3 & 7 & 3 \end{bmatrix} & \begin{array}{c} H \\ S \end{array} \end{array}$$

Both dealers want to increase their 2015 sales by 50% by 2016. Construct a new matrix $E=1.5B$. Explain why this matrix represents the planned sales figures for 2016.

$$E = 1.5 \times B$$

$$\begin{array}{r} 100\% \\ +\ 50\% \\ \hline 150\% \end{array} \rightarrow \div 100 = 1.5$$

$$E = 1.5 \times \begin{bmatrix} 26 & 38 & 16 \\ 35 & 41 & 12 \end{bmatrix}$$

$$= \begin{bmatrix} 39 & 57 & 24 \\ 53 & 62 & 18 \end{bmatrix}$$