Matrix powers

Thursday, 23 April 2020

🜟 By

By the end of the lesson I would hope that you have an understanding and be able to apply to questions the following concepts:

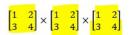
· What does it mean to have a matrix power

7:35 PM

· How do we raise a matrix to a power

RECAP

This is a relatively quick lesson building on some of the previous work. Knowing what we do about multiplying matrices .. What if we had to do the following?





Or

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

No one ... in their right mind would want to do that by hand!

We can use the calculator, but it's going to take a long time to type in all those matrices.

So ... we can use powers!

Raising a matrix to a power

When we raise something to a power, we are really just multiplying it by itself a number of times.

Hence:

Is the same as:

 $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

 $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

2° = 2×2 6³= 6×6×6

And:

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

Is the same as:

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \bigotimes \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \bigotimes \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

How is this helpful?

All in good time!

Example

The following example has been extracted, with permission, from the Cambridge Further Maths Units 3 and 4 Textbook

If
$$A=\begin{bmatrix}1&0\\2&-1\end{bmatrix}$$
, $B=\begin{bmatrix}-1&1\\2&1\end{bmatrix}$ and $C=\begin{bmatrix}0&1\\1&1\end{bmatrix}$, determine:

- a $2A + B^2 2C$
- **b** $(2A B)^2 C^2$
- $AB^2 3C^2$

a)
$$\begin{bmatrix} s - 2 \\ 2 - 1 \end{bmatrix}$$