Matrix powers

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By the end of the lesson I would hope that you have an understanding and be able to apply to questions the following concepts:

- What does it mean to have a matrix power
- How do we raise a matrix to a power

RECAP

This is a relatively quick lesson building on some of the previous work.
Knowing what we do about multiplying matrices .. What if we had to do the following?

$$\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}\times\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}\times\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}$$

Or ....

$$\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}\times\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}\times\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}\times\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}$$

No one ... in their right mind would want to do that by hand!

We can use the calculator, but it's going to take a long time to type in all those matrices.

So ... we can use powers!

Raising a matrix to a power

When we raise something to a power, we are really just multiplying it by itself a number of times.

Hence:

$$\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}^2$$

$2^2 = 2\times 2$

$6^3 = 6\times 6\times 6$

Is the same as:

$$\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}\times\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}$$

And:

$$\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}^3$$

Is the same as:

$$\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}\times\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}\times\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}$$

How is this helpful?

All in good time!

Example

The following example has been extracted, with permission, from the Cambridge Further Maths Units 3 and 4 Textbook.

If $A = \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 1 \\ 2 & 1 \end{bmatrix}$ and $C = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$, determine:

a $2A + B^2 - 2C$

b $(2A - B)^2 - C^2$

c $AB^2 - 3C^2$

a) $\begin{bmatrix} 5 & -2 \\ 2 & -1 \end{bmatrix}$

b) $\begin{bmatrix} 6 & -1 \\ -1 & 5 \end{bmatrix}$

c) $\begin{bmatrix} 0 & -3 \\ 3 & -9 \end{bmatrix}$