Defining sine, cosine and tangent

Sunday, 20 October 2019 12:19 pm

★ By the end of the lesson I would hope that you have an understanding (and be able to apply to questions) the following concepts:

- Know what it means by finding the sine, cosine and tangent of an angle
- Know that this applies to right angled triangles only
- Know the trigonometric ratios to help us find the size of angles
- Know the trigonometric ratios to help us find the size of missing sides

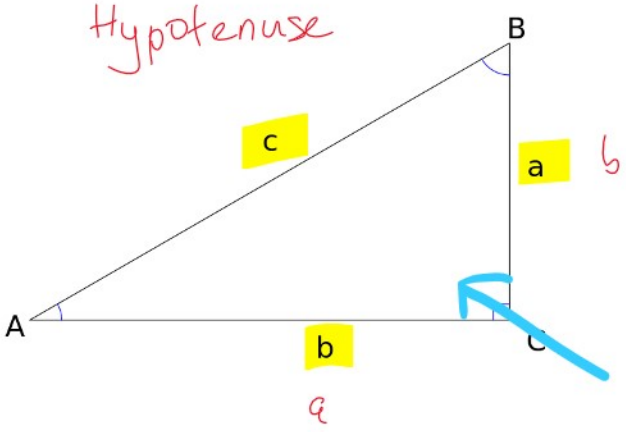
RECAP

This is the start of a new section of work, but builds on the work which everyone will have done in Year 9.
Trigonometry is awesome!
It has three formulae which can be used to find missing side lengths and missing angles in right angled triangles.
It uses the calculator to help you find the missing values!
The hardest part of Trigonometry is making sure you know how to swap fractions around.

RECAP: Trigonometry

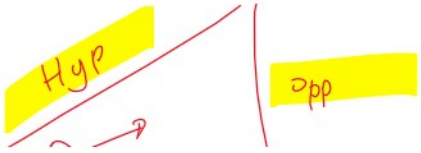
This only works with Right Angled Triangles!
We have met right angled triangles before in Pythagoras' Theorem which helped us find the size of missing sides.

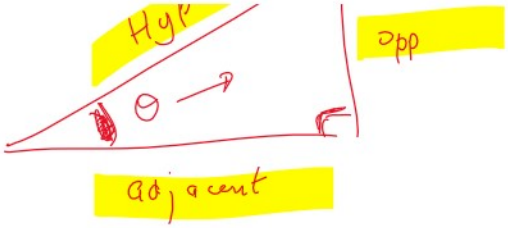
$c^2 = a^2 + b^2$



Trigonometry labels the sides using different terms.
The sides are all named by referencing one main angle.

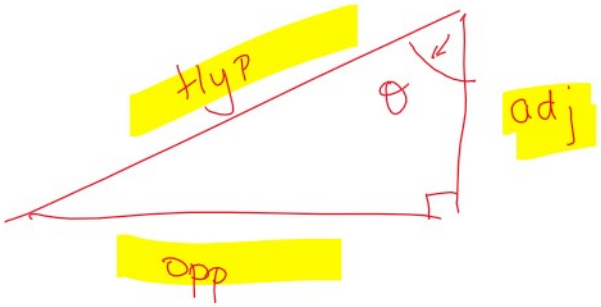
~~Opp~~
adjacent





Hence, the sides are named as Opposite, Adjacent and Hypotenuse.

Note: The hypotenuse is ALWAYS the longest side and always the side which is opposite the right angle.



If we use a different angle, then the sides will have different names.

Note: Always mark the Hypotenuse, then the opposite. The Adjacent will be the only side left.

SOHCAHTOA

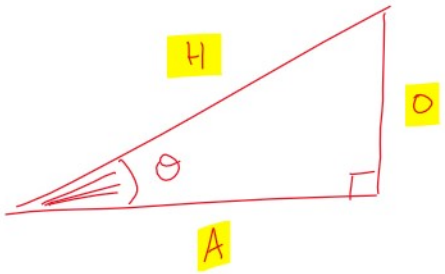
The above word would have been much simpler to learn that the following:

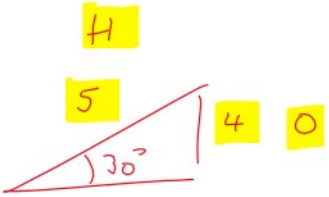
Silly
Old
Harry
Caught
A
Herring
Trawling
Off
America

But what does it mean?
It's all about helping us to remember the three trigonometric ratios which we can use to answer questions.

SOH CAH TOA

$\sin\theta = \frac{O}{H}$ $\cos\theta = \frac{A}{H}$ $\tan\theta = \frac{O}{A}$





So, the three trigonometric ratios you need to remember are:

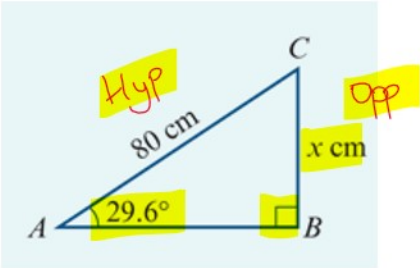
$$\sin\theta = \frac{opposite}{hypotenuse} \qquad \cos\theta = \frac{adjacent}{hypotenuse} \qquad \tan\theta = \frac{opposite}{adjacent}$$

Examples

The following examples are used, with permission, from the Cambridge Further Mathematics Units 3 and 4 textbook.

Example 1

Find the value of x, correct to two decimal places.



$\sin\theta = \frac{Opp}{Hyp}$

$\sin 29.6 = \frac{x}{80}$

$x = 39.52 \text{ cm}$

SOH CAH TOA

S O
H

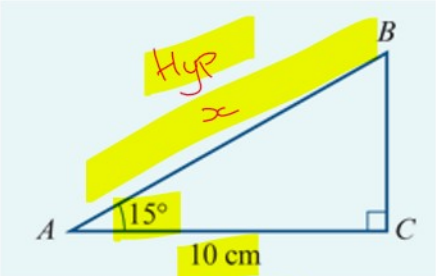
$\sin 29.6 = \frac{x}{80}$

$x = 80 \times \sin 29.6$

$= 39.52 \text{ cm}$

Example 2

Find the length of the hypotenuse, correct to two decimal places.



Adj

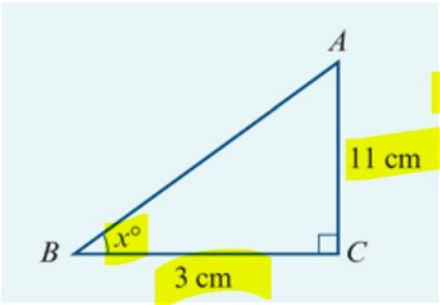
S C T
O A O
H H A

$\cos\theta = \frac{Adj}{Hyp}$

$\cos 15 = \frac{10}{x}$

$x = 10.35 \text{ cm}$

Example 3
Find the magnitude of $\angle ABC$.



opp

adj

SOH CAH TOA

$\tan\theta = \frac{opp}{adj}$

$\tan x = \frac{11}{3}$

$x = 74.74^\circ$

Interesting things to note about Sine and Cosine angles

We always have been taught that there is only one angle for each sine, cosine and tangent value.
This isn't actually true!

Let's explore using the calculator ...

$\sin 30^\circ = \sin 150^\circ$

$\sin 40^\circ = \sin 140^\circ$

$\cos 30^\circ = -\cos 150^\circ$

$\cos 50^\circ = -\cos 130^\circ$

$\sin 40^\circ = \sin 140^\circ$

$\sin 20^\circ = \sin 160^\circ$

$\sin 60^\circ = \sin 120^\circ$

$\sin\theta = \sin(180-\theta)$

$\cos 50^\circ = -\cos 130^\circ$

$\cos 60^\circ = -\cos 120^\circ$

$\cos\theta = -\cos(180-\theta)$

We need to remember the following

$$\sin\theta^\circ = \sin(180 - \theta^\circ)$$

$$\cos\theta = -\cos(180 - \theta^\circ)$$