Combining geometric growth and decay

Sunday, 24 March 2019 1:48 pm

★ By the end of the lesson I would hope that you have an understanding and be able to apply to questions the following concepts:

- How to generate a sequence from a recurrence relation
- How to model population changes using a recurrence relation

RECAP

This lesson is the first in a new chapter of lessons.
We are building on the knowledge gained from Modelling growth and decay using recursion.
We have already used the idea what we can create numbers in a sequence using the following formula:

$$V_0 = starting\ value, \qquad V_{n+1} = R \times V_n \pm D$$

V_0 R

V_n V_{n+1} $\pm D$

Where:

- V_{n+1} is the next term in the sequence
- R is the multiplier (which can be greater and smaller than 1)
- V_n is the previous (or current) term
- D is the value of the fixed rate payments which are made

$V_{n+1} = R \times V_n + D$

$V_{n+1} = R \times V_n - D$

Note the use of the $\pm$ which means that the equation is representing two equations.
One for **growth** and one for **decay**.

The positive would model GROWTH.
The negative would model DECAY.

Example

The following example has been extracted, with permission, from the Cambridge Further Maths Units 3 and 4 Textbook.

Write down the sequence generated by the recurrence relation

$V_0 = 3, \quad V_{n+1} = 4V_n - 1$

$V_{n+1} = 4 \times V_n - 1$

$V_0 = 3$

$V_1 = 3 \times 4 - 1$

$= 11$

$V_2 = 4 \times 11 - 1$

$= 43$

$V_3 = 4 \times 43 - 1$

$= 171$

Example

The following example has been extracted, with permission, from the Cambridge Further Maths Units 3 and 4 Textbook.

The number of trout in a fish farm pond after n months, Tn, can be modelled using the recurrence relation:

$T_0 = 10000, T_{n+1} = 1.1 \times T_n - 3000$

- *Use the recurrence relation to determine the number of trout in the pond after 2 months.*

$T_0 = 10\,000$

$T_1 = 10\,000 \times 1.1 - 3000 = 8000$

$T_2 = 80\,000 \times 1.1 - 3000 = 5800$

- *After how many months will there be no trout left in the pond?*

$T_3 = 3380$

$T_4 = 718$

$T_5 = -2210.2$