Areas, volumes and similarity

Sunday, 20 October 2019 12:18 pm

★ By the end of the lesson I would hope that you have an understanding (and be able to apply to questions) the following concepts:

- Know how to find the areas and volumes of similar shapes using a scale factor

RECAP

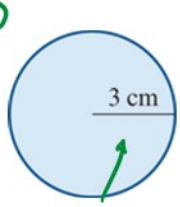
This is the final lesson in this series for the Further Mathematics Units 3 and 4 chapter.
We are going to build on the knowledge we have acquired on:

Finding the volume and surface areas of some of the following shapes

- Cylinder
- Pyramid
- Sphere
- Cone
- Composite shapes
- Cuboid

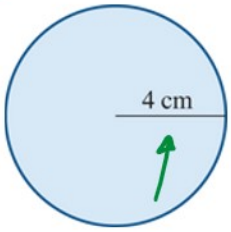
Areas of Similar Circles

Let's look at what happens to the areas of similar circles by considering their scale factor of enlargement.

① 

$Scale\ Factor\ of\ Enlargement = \frac{4}{3}$

$k = \frac{4}{3}$

② 

$Area = \pi \times r^2$

$= \pi \times 3^2$

$= 9\pi$

$Area = \pi \times r^2$

$= \pi \times 4^2$

$= 16\pi$

$$\text{Scale factor area} = \frac{16\pi}{9\pi} = \frac{16}{9} = \frac{4^2}{3^2} = \left(\frac{4}{3}\right)^2$$

$$= k^2$$

$= K^2$

Areas of Similar Rectangles

Let's look at what happens to the areas of similar rectangles by considering their scale factor of enlargement.



Scale Factor of Enlargement = 2

$K = 2$



$A = 2 \times 3$

$= 6\ cm^2$

$A = 6 \times 4$

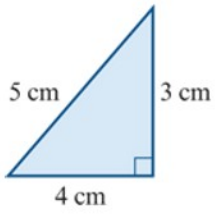
$= 24\ cm^2$

$$\text{Scale factor Area} = \frac{24}{6} = 4 = (2)^2$$

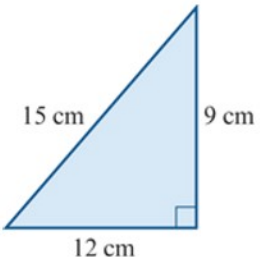
$= K^2$

Areas of Similar Triangles

Let's look at what happens to the areas of similar triangles by considering their scale factor of enlargement.



$Scale\ Factor\ of\ Enlargement = 3$



Area $= \frac{1}{2}bh$

$= \frac{1}{2} \times 4 \times 3$

$= 2 \times 3$

$= 6\ cm^2$

Area $= \frac{1}{2}bh$

$= \frac{1}{2} \times 12 \times 9$

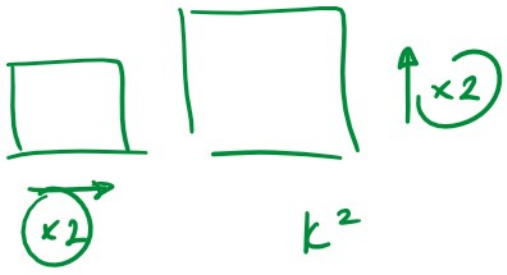
$= 6 \times 9$

$= 54\ cm^2$

Scale Factor Area $= \frac{54}{6} = 9 = (3)^2$

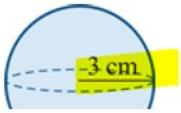
When scaling areas we see the following (where k is the scale factor of enlargement):

$$Area\ of\ similar\ shape = k^2 \times area\ of\ original\ shape$$

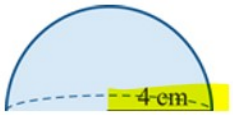


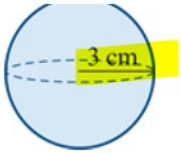
Volumes of Similar Spheres

Let's look at what happens to the volumes of similar spheres by considering their scale factor of enlargement.

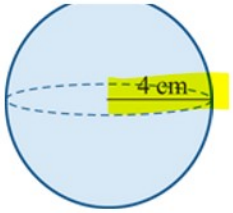


$Scale\ Factor\ of\ Enlargement = \frac{4}{3}$





$Scale\ Factor\ of\ Enlargement = \frac{4}{3}$



$Volume = \frac{4}{3}\pi \times r^3$

$= \frac{4}{3} \times \pi \times 3^3$

$= 4 \times \pi \times 9$

$= 36\pi\ cm^3$

64 | 4 | 256 | 1

$Volume = \frac{4}{3}\pi \times r^3$

$= \frac{4}{3} \times \pi \times 4^3$

$= \frac{4}{3} \times \pi \times 64$

$= \frac{256\pi}{3}$

Scale Factor Volume $= \frac{256\pi}{3} \times \frac{1}{36\pi}$

$= \frac{256}{108}$

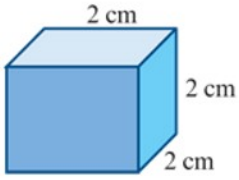
36 | 3 | 108

$\therefore \frac{64}{27} = \left(\frac{4}{3}\right)^3$

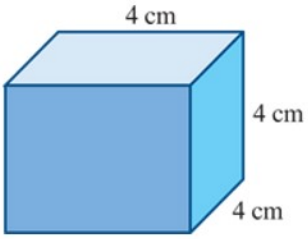
k^3

Volumes of Similar Cubes

Let's look at what happens to the volumes of similar cubes by considering their scale factor of enlargement.



$Scale\ Factor\ of\ Enlargement = 2$



$V = 8\ cm^3$

$V = 4 \times 4 \times 4$

$= 16 \times 4$

$= 16 \times 4$

$= 64\ cm^3$

Volume Scale factor $= \frac{64}{8} = 8 = (2)^3$

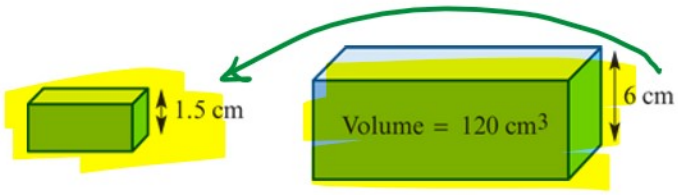
When scaling volumes we see the following (where k is the scale factor of enlargement):

$$Volume\ of\ similar\ shape = k^3 \times\ volume\ of\ original\ shape$$

Examples

The following examples have been extracted, with permission, from the Cambridge Further Mathematics Units 3 and 4 textbook.

The two cuboids shown are similar solids.
The height of the larger cuboid is 6 cm.
Its volume is 120 cm^3.
The height of the smaller cuboid is 1.5 cm.
Determine its volume.



$\frac{1.5}{6} = \frac{1}{4}$

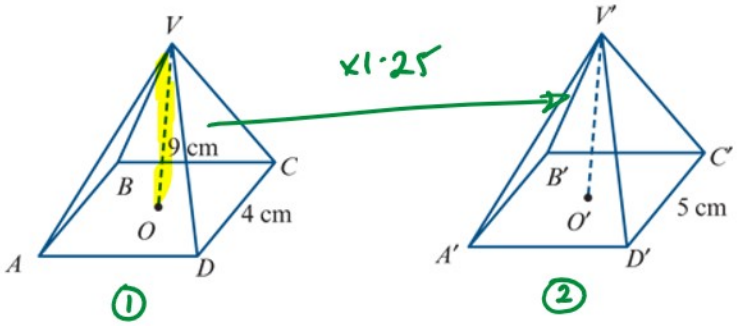
Vol sim $=$ (scale factor)^3 x orig

$= \left(\frac{1}{4}\right)^3 \times 120$

$$= \left(\frac{1}{4}\right)^3 \times 120$$

$$= 1.875\ cm^3$$

The two square pyramids shown are similar with a base dimensions 4 and 5 cm, respectively. The height of the first pyramid is 9 cm and its volume is 48 cm^3. Find the height and volume of the second pyramid.



$$k = \frac{s}{4} = 1.25$$

Height = $9 \times 1.25 = 11.25$ cm.

Volume = $k^3 \times$ volume original

$$= (1.25)^3 \times 48$$

$$= 93.75\ cm^3$$