Arc length, area of a sector, area of a segment

★ By the end of the lesson I would hope that you have an understanding and be able to apply to questions the following concepts:

- Know what an arc length is and how to find it
- Know what a sector is and how to find its areas
- Know what a segment is and how to find its area

RECAP

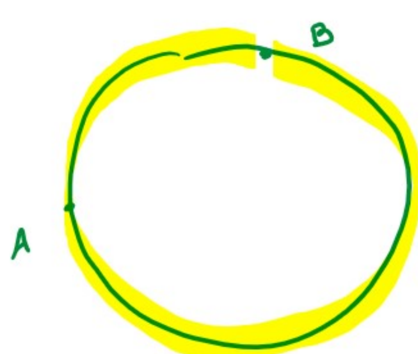
This is the first lesson in a series on Spherical Geometry.
It's really rather interesting and will look at not only finding lengths and areas of circles (and their parts) but how this applies to time zones and finding our way around the earth.

Firstly, let's remind ourselves about the important features of a circle:

Radius:

Diameter:

Arc (Major and Minor)



Barry is at it again with his language!

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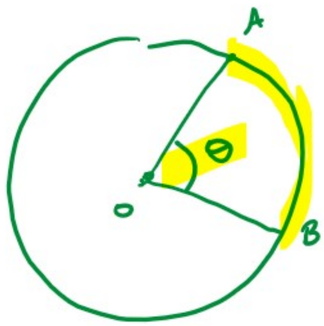
Well, here we are again with the word **subtend**.
Probably not one you can guess!

Subtend: This is the angle at the centre of a circle which is described by an arc length.



Finding the Arc Length using Maths you already know

We know the circumference of a **full** circle is given by:

$$Circumference = 2 \times \pi \times r$$

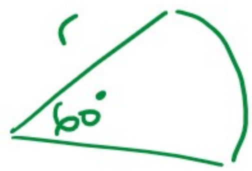
How would we find the circumference of a quarter of a circle?

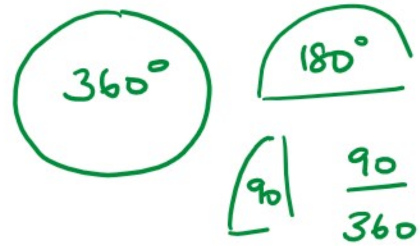
$$C = \frac{2 \times \pi \times r}{4}$$

How would we find the circumference of a half a circle?

$$C = \frac{2\pi r}{2}$$

How would we find the circumference of a circle which has a subtended angle of 60°?





$180/360 = \frac{1}{2}$

$\frac{90}{360} = \frac{1}{4}$

$$c = \frac{60}{360} \times 2\pi r$$

Formula for the length of an arc:

$$Arc\ length = \frac{\theta}{360} \times 2\pi r$$

Fraction of whole circle.

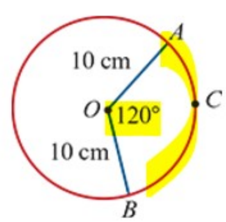
$$\cancel{Arc\ length = \frac{\pi r\theta}{180}}$$

Example

The following examples have been extracted, with permission, from the Cambridge Further Mathematics Units 3 and 4 textbook.

In this circle, centre O, radius length 10 cm, the angle subtended at O by arc ACB has magnitude.

Find the length of the arc ACB, correct to one decimal place.



$$Arc\ length = \frac{120}{360} \times 2 \times \pi \times 10$$

$$= 20 \cdot 9\,cm$$

$= 20 \cdot 9\text{cm}$

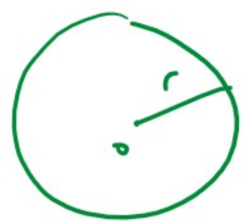
Area of a Sector

We can use the same idea to find the area of a sector. But what is a sector?

It's a pie slice!

$r^2 = r \times r$

We know the area of a whole circle is:



$$Area = \pi r^2$$

So it follows that is we have a section of a circle (**sector**) then, so long as we know the size of the angle of the slice, we can find the area of the sector.

$$Area\ of\ a\ sector = \frac{\theta}{360} \times \pi r^2$$

$$Arc = \frac{\theta}{360} \times 2\pi r$$

$$\frac{60}{360} \times \pi r^2$$

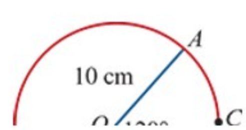
Example

The following examples have been extracted, with permission, from the Cambridge Further Mathematics Units 3 and 4 textbook.

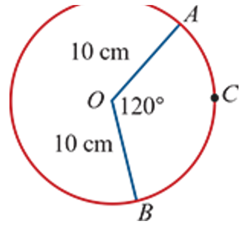
In this circle, centre O, radius length 10 cm, the angle subtended at O by arc ACB has magnitude 120°.

Find:

- the area of the minor sector AOB
- the area of the major sector AOB.



- the area of the minor sector AOB
- the area of the major sector AOB.



Give your answer correct to two decimal places.

AOB $\quad$ Area $= \dfrac{120^\circ}{360^\circ} \times \pi\ 10^2$

$= \underline{104 \cdot 72\ cm^2}$

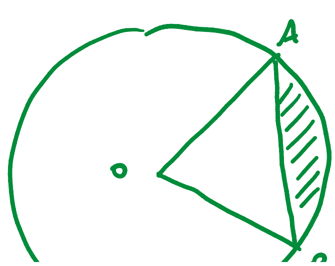
AOB $\quad$ Area $= \dfrac{240}{360} \times \pi \times 10^2$

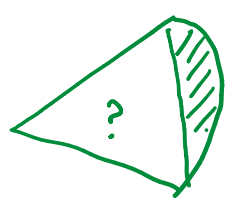
$= \underline{209.44\ cm^2}$

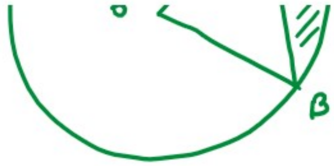
Finding the area of a segment

What is this segment I hear you ask?!

An example of a segment is shown below:







To find the area of a segment we need to find the area of a sector and subtract from it the area of the triangle, which will leave the area of the sector.

We already know the area of a sector can be found using:

$$Area\ of\ a\ sector = \frac{\theta}{360} \times \pi r^2$$

We also know, from a previous lesson, that we can find the area of a triangle if we know two side lengths and the included angle using:

$$Area\ of\ a\ triangle = \frac{1}{2}\ ab\ sin\ C$$

Where a and b are the side lengths (radii) and C is the size of the included angle

$$A = \frac{1}{2} r^2 \sin\theta$$

Hence we can combine the formulae to become:

$$\text{Area of segment} = \frac{\theta}{360} \times \pi r^2 - \frac{1}{2} r^2 \sin\theta$$

Example

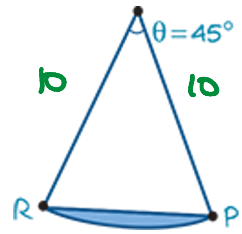
The following examples have been extracted, with permission, from the Cambridge Further Mathematics Units 3 and 4 textbook.

Find the area A of the segment that is shaded blue.
The radius of the circle is 10 cm.

Units 3 and 4 textbook.

Find the area A of the segment that is shaded blue.
The radius of the circle is 10 cm.

Give your answer, correct to two decimal places.



$$A = \frac{\theta}{360} \times \pi r^2 - \frac{1}{2} r^2 \sin\theta$$

$$= \frac{45}{360} \times \pi \times 10^2 - \frac{1}{2} \cdot 10^2 \cdot \sin 45$$

$$= \underline{3.91 \text{ cm}^2}$$