Annuities

Sunday, 24 March 2019 12:50 pm

★ By the end of the lesson I would hope that you have an understanding and be able to apply to questions the following concepts:

- Know how to use the Financial Solver on your CAS to answer questions relating to annuities
- Understand what it means to be an annuity
- Know what an amortisation table is and how to read it
- Understand the language used for annuities
  - V_0 as the principal
  - r as the rate of interest
  - D as the regular payment per compounding period

RECAP:

In the previous lessons we have been looking at how to use the financial solver to help us deal with reducing balance loans and interest-only loans.
This has been very exciting.
There are two more ways of dealing with financial maths and in this lesson we are going to look at an **annuity**.

WOT IS AN ANNUITY?

Other than a really stupid word to try and spell …
This is where you give the bank money (-ve) and, over a period of time, it gets paid back to you in a fixed amount each month until the money has run out.

Generally speaking, you will know how much you are investing. You will know the rate of interest, and you will then be able to find out how much money per month you can pay out (for a certain period of time) until the money all runs out.

This is very similar to the Australian Pension Scheme.

Recurrence Relation for an Annuity

We love recurrence relations!

With annuities we need to know the following:

- r : the interest rate per compounding period
- D : the payment received
- V_0 : the principal invested

We can then model an annuity using the following recurrence relation:

$$V_0 = principal, \qquad V_{n+1} = R \times V_n - D$$

$$R = 1 + \frac{r}{100}$$

Example:
This example is extracted from the *Cambridge Further Mathematics Units 3 and 4 Textbook:*

Reza plans to travel overseas for 6 months. He invests $12000 in annuity that earns interest at the rate of 6% per annum, providing him with a monthly income of $2035 per month for 6 months.

Model this annuity using a recurrence relation of the form

$V_0 = the\ principal, V_{n+1} = RV_n - D$

where V_n is the value of the annuity after n payments have been received.

$V_0 = the\ principal, V_{n+1} = kV_n - D$

where V_n is the value of the annuity after n payments have been received.

$V_0 = 12\,000$

$r = 6\%$ p.a.

$r = \frac{6\%}{12}$ p.m

$= \frac{1}{2}\%$ p.m

$R = 1 + \frac{0.5}{100} = 1.005$

$V_0 = 12\,000, \quad V_{n+1} = 1.005 \times V_n - 2035$

WOT ABOUT THE CAS?

I know!
This is a CAS course right …
So, let's use the CAS to answer some questions about Reza's annuity …

Reza's annuity can be modelled by the recurrence relation

$V_0 = 12000, V_{n+1} = 1.005V_n - 2035$

where V_n is the value of the annuity after n payments have been received.

a. Use your calculator to determine recursively the value of the annuity after Reza has received three payments from the annuity.
b. Is the annuity fully paid out after six monthly payments have been made? If not, how much will the last payment have to be to ensure that the annuity terminates after 6 months?

$V_0 = 12000, \quad V_{n+1} = V_n \times 1.005 - 2035$

a) \$6045.33

b) \$0.88

\$2035.88

Amortisation tables for annuities

Sounds like something from a horror movie …
But … it's actually just a table.
We can model this using excel …

Can the Financial Solver handle annuities?

Of course!
And it's a breeze …

Example:
Joe invests $200000 into an annuity, paying 5% compound interest per annum, compounding monthly.

a. If he wishes to be paid monthly payments for 10 years, how much will he receive each month?
b. If he receives a regular monthly payment of $3000, how long will the annuity last? Give your answer correct to the nearest month.
c. What interest rate, correct to one decimal place, would allow Joe to withdraw $2500 each month for 10 years?

N: 10 x 12

I: 5

PV: −200 000

PMT: −

FV: 0

P/Y: 12

C/Y: 12

a) $2121.31

b) 78 months!

c) 8.7%

N:

I:

PV:

PMT:

FV:

P/Y:

C/Y: