Angles of elevation and depression, bearings, and triangulation

Sunday, 19 April 2020 10:24 am

★ By the end of the lesson I would hope that you have an understanding (and be able to apply to questions) the following concepts:

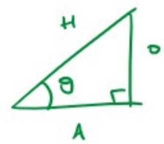
- Understand what angles of elevation and depression are
- Understand what bearings are
- Understand the concept of triangulation and how to apply it to questions

RECAP

In previous lessons we have looked at how to find angles and side lengths of both right-angled and non right-angled triangles. We have used the concepts of trigonometry (for right-angled triangles) and the sine and cosine rules for non right-angled triangles.

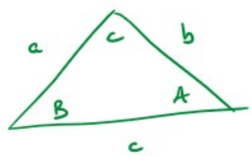
Right Angled Triangles and Trigonometry:

$$\sin\theta = \frac{opposite}{hypotenuse} \qquad \cos\theta = \frac{adjacent}{hypotenuse} \qquad \tan\theta = \frac{opposite}{adjacent}$$



Sine Rule:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$



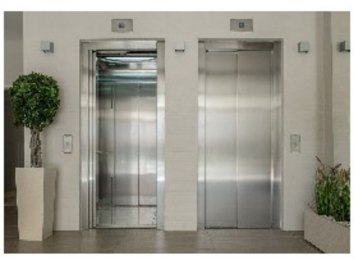
Cosine Rule:

$$a^2 = b^2 + c^2 - 2 \times b \times c \times \cos A$$

We are now going to look at how to apply the above to "Real World" situations.

Angles of elevation and depression

Elevators go up.
OK ... they also go down ... but that's not going to work for my example!

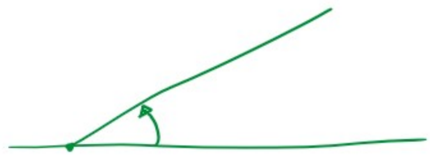


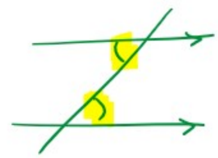
And when I'm feeling a bit depressed ... I'm feeling a bit down.

Beyond Blue

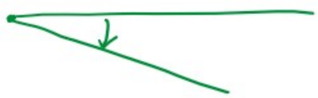
This is how I remember angles of **elevation** and **depression**

Angles of Elevation start from the horizontal and go up.





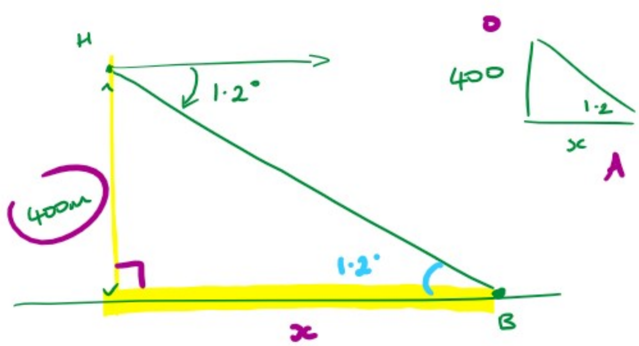
Angles of depression start from the horizontal and go down.



Examples of how to find angles of elevation and depression

The following examples have been extracted, with permission, from the Cambridge Further Mathematics Units 3 and 4 textbook.

The pilot of a helicopter flying at 400 m observes a small boat at an angle of depression of 1.2°. Draw a diagram and calculate the horizontal distance of the boat to the helicopter, correct to the nearest 10 metres.



$$\tan\theta = \frac{Opp}{Adj}$$

$$\tan 1.2 = \frac{400}{x}$$

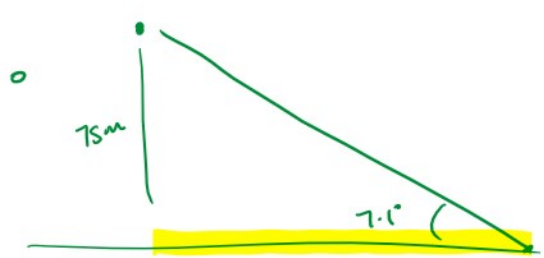
$$x = \frac{400}{\tan 1.2}$$

$$x = 19095.8 \text{ m}$$

$$= \underline{19100 \text{ m}}.$$

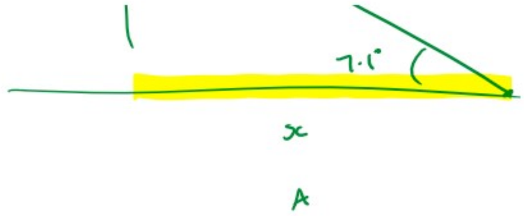
The light on a cliff-top lighthouse, known to be 75 m above sea level, is observed from a boat at an angle of elevation of 7.1°. Draw a diagram and calculate the distance of the boat from the lighthouse, to the nearest metre.

$$\tan 7.1 = \frac{75}{x}$$



$$x = \frac{75}{\tan 7.1}$$

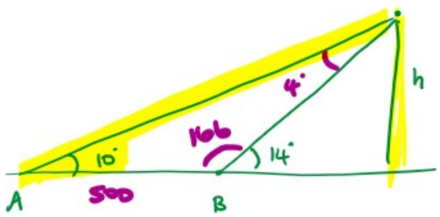
$$= 602.14 \text{ m}$$

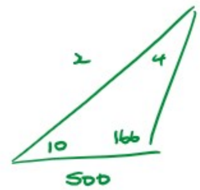


$= 602.14 \text{ m}$

$= \underline{\underline{602 \text{ m.}}}$

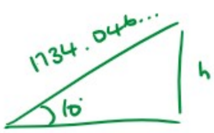
From a point A, a man observes that the angle of elevation of the summit of a hill is 10°. He then walks towards the hill for 500 m along flat ground. The summit of the hill is now at an angle of elevation of 14°. Draw a diagram and find the height of the hill above the level of A, to the nearest metre.





$$\frac{a}{\sin A} = \frac{b}{\sin B} \Rightarrow \frac{500}{\sin 4} = \frac{x}{\sin 166}$$

$$x = \underline{1734.046\ldots \text{ m}}$$



$$\sin 10° = \frac{h}{1734.046\ldots}$$

$$h = 301.11\ldots \text{ m}$$
$$= 301 \text{ m}$$

Bearings - How bears cross the water ...

I couldn't find a decent picture!

A bearing consists of two pieces of information:

- An angle (from the north line measured clockwise)
- A distance.

Most times, we are interested in just the angle.

Bearings are always expressed with **three digits**.

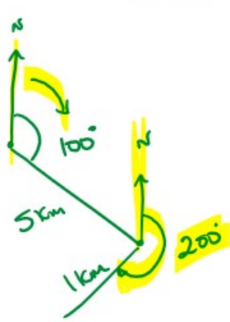
The most important thing with bearings it knowing what the word "from" means.

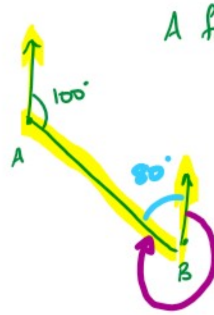
This is important!

090°T

B from A 100°

A from B ?





$360° - 80°$

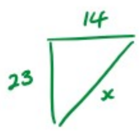
$360° - 80°$

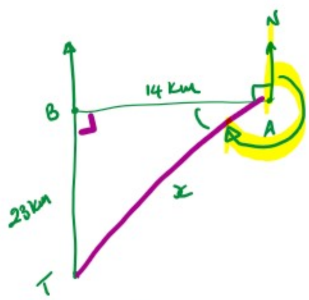
$= 280°T$

Examples of how to express and find bearings

The following examples have been extracted, with permission, from the Cambridge Further Mathematics Units 3 and 4 textbook.

The road from town A runs due west for 14 km to town B. A television mast is located due south of B at a distance of 23 km. Draw a diagram and calculate the distance of the mast from the centre of town A, to the nearest kilometre. Find the bearing of the mast from the centre of the town.





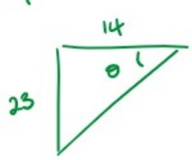
$x^2 = 14^2 + 23^2$

$= 725$

$x = \sqrt{725}$

$x = 26.93\text{km}$

$x = \underline{27\text{ km}}$



$\tan\theta = \frac{23}{14}$

$\theta = 58.67...°$

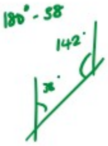
$\text{Bearing} = 360° - 90° - 58.67..°$

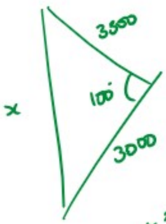
$= 211.32°$ $\quad \underline{211°T}$

A yacht starts from a point A and sails on a bearing of 038° for 3000 m. It then alters its course to a bearing of 318°, and after sailing for 3300 m it reaches a point B.

- Find the distance AB, correct to the nearest metre.
- Find the bearing of B from A, correct to the nearest degree.

$180° - 38$

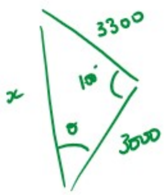




$$x^2 = 3300^2 + 3000^2 - 2 . 3300 . 3000 . \cos 100^\circ$$

$$x^2 = 23328253 \cdot 92$$

$$x = \underline{4830 \text{ M}}$$



$$\frac{3300}{\sin\theta} = \frac{x}{\sin 100}$$

$$\frac{3300 \times \sin 100}{x} = \sin\theta$$

$$\sin\theta = 0 \cdot 67285 \ldots$$

$$\theta = 42 \cdot 288 \ldots$$

Slither = $42^\circ \cdot 288 \ldots - 38$

$= 4 \cdot 288$

Bearing $= 360' - 4 \cdot 288$

$= \underline{356^\circ}$

Triangles and knowing where you are in the world

I love watching Blacklist at the moment. They are forever "triangulating" someone's position from their cell phone.
This is beyond this part of the course, but we can use the idea to find objects using triangles.

Two points, A and B, are on opposite sides of a lake so that the distance between them cannot be measured directly.
A third point, C, is chosen at a distance of 100 m from A and with angles BAC and BCA of 65° and 55°, respectively.

Calculate the distance between A and B, correct to two decimal places.

$180^\circ - \underline{120^\circ} = 60^\circ$

$$\frac{x}{\sin 55} = \frac{100}{\sin 60}$$

$$x = \frac{100 \times \sin 55}{\sin 60}$$

$$= 94 \cdot 5875$$

$$= \underline{94 \cdot 59 \text{ M}}$$