Relations, domain and range

Sunday, 25 March 2018 9:14 pm

By the end of the lesson I would hope that you have understanding and knowledge of the following key concepts by the end of the lesson:

- Know what it means to be an ordered pair
- Know what it means by a relation
- Know what it means by domain
- Know what it means by range
- Know how to write domain and range in **Interval Notation**
- Know how to write a relation in set notation

RECAP:

In a previous lesson we started to build the language we need to be able to confidently talk about and describe graphs.
We have, previously, looked at using inequalities to describe sets of solutions.
We now know about something called Interval Notation
This lesson is going to introduce some more concepts to you which are critical to the rest of the course.

RECAP: Interval Notation

We can express sets of numbers using a simple form of notation which includes **square** and **curved** brackets.
Examples are given below.
Note: Infinity ∞ is an idea or construct. It doesn't exist and hence cannot have a square bracket!

$x \geq 3$ $[3, \infty)$ $-1 \leq x \leq 10$

$x < 10$ $(-\infty, 10)$ $[-1, 10]$

Barry ALERT: New language

As is normal, Mathematics would be awesome without Barry.
He's decided that he wants to introduce a two new terms:

Ordered Pair:

$(0,4)$ $(2,6)$ $(10,-3)$

Relation:

$y = x^2$

$y = 3x - 2$

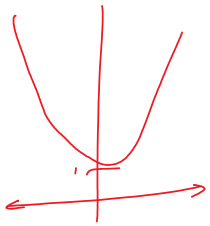
$y = x^3 - 2x^2 + x + 4$

With that covered … we now need to look at graphs and how we can describe how the graph is created in terms of x-values and y-values.

Domain and Range

A graph is nothing more than a relationship between x and y values.
We use a formula to describe these values!

Example: $y = x^2 + 1$



We know that the x-values we put into the formula create the y-values which come out!
But it's interesting to note that WE CHOOSE THE x-VALUES

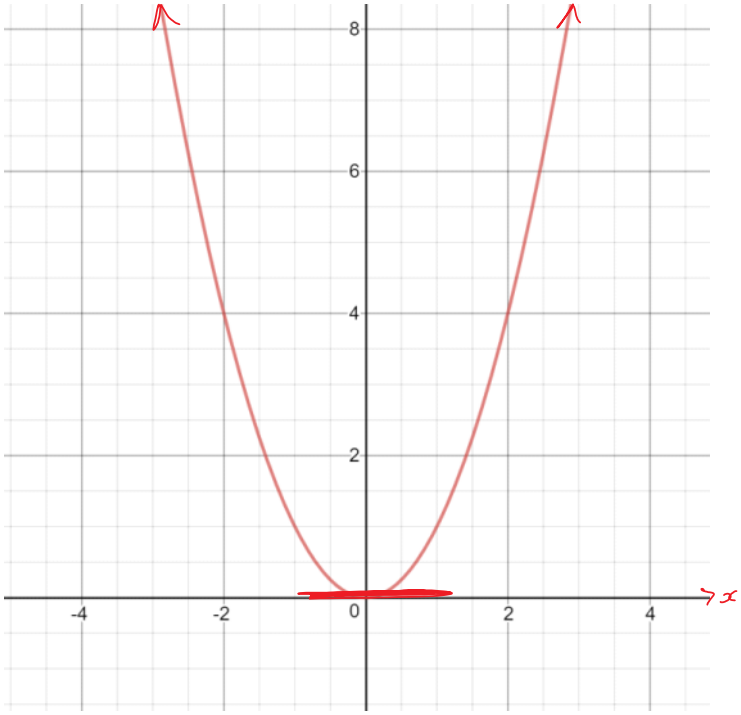
We can choose to allow all values into the equation, or we can limit the values to certain numbers only.
This is called limiting the domain and we will come to that in a moment.

DOMAIN: The possible x-values which can be placed into an equation.
RANGE: The possible y-values which are created as a result of putting in some x-values.

It can't be explained any more clearly.

Let's look at some examples

The graph of $y = x^2$



~~Domain $(-\infty, \infty)$~~

$\mathbb{R}$

$x \in (-\infty, \infty)$

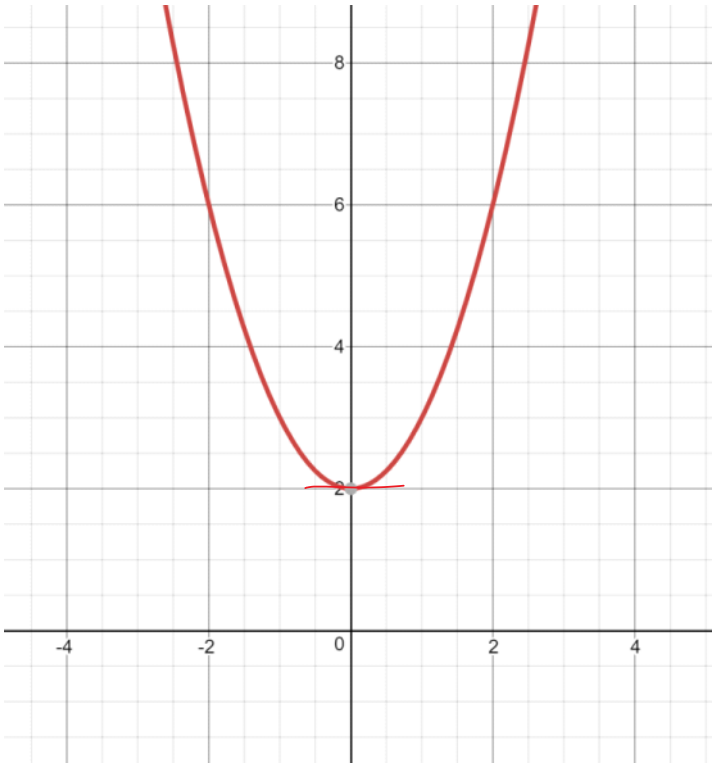
$x \in \mathbb{R}$

~~Range $[0, \infty)$~~

$y \in [0, \infty)$

$y \in \mathbb{R}^+ \cup \{0\}$

The graph of $y = x^2 + 2$



Domain

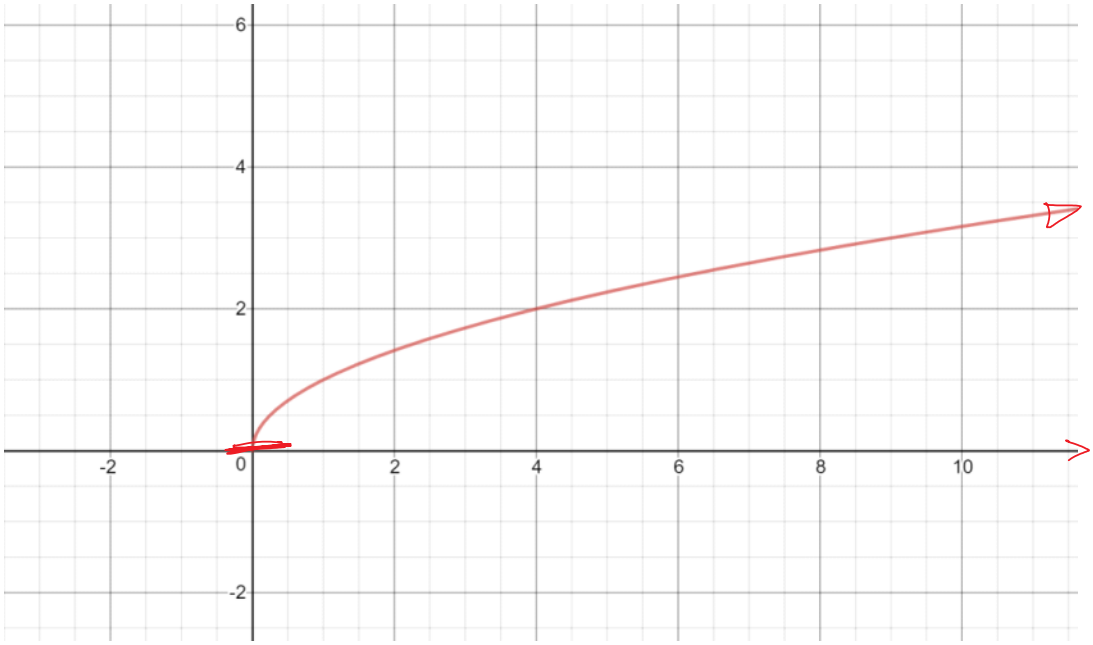
$x \in (-\infty, \infty)$

$x \in \mathbb{R}$

Range

$y \in [2, \infty)$ ✓

The graph of $y = \sqrt{x}$



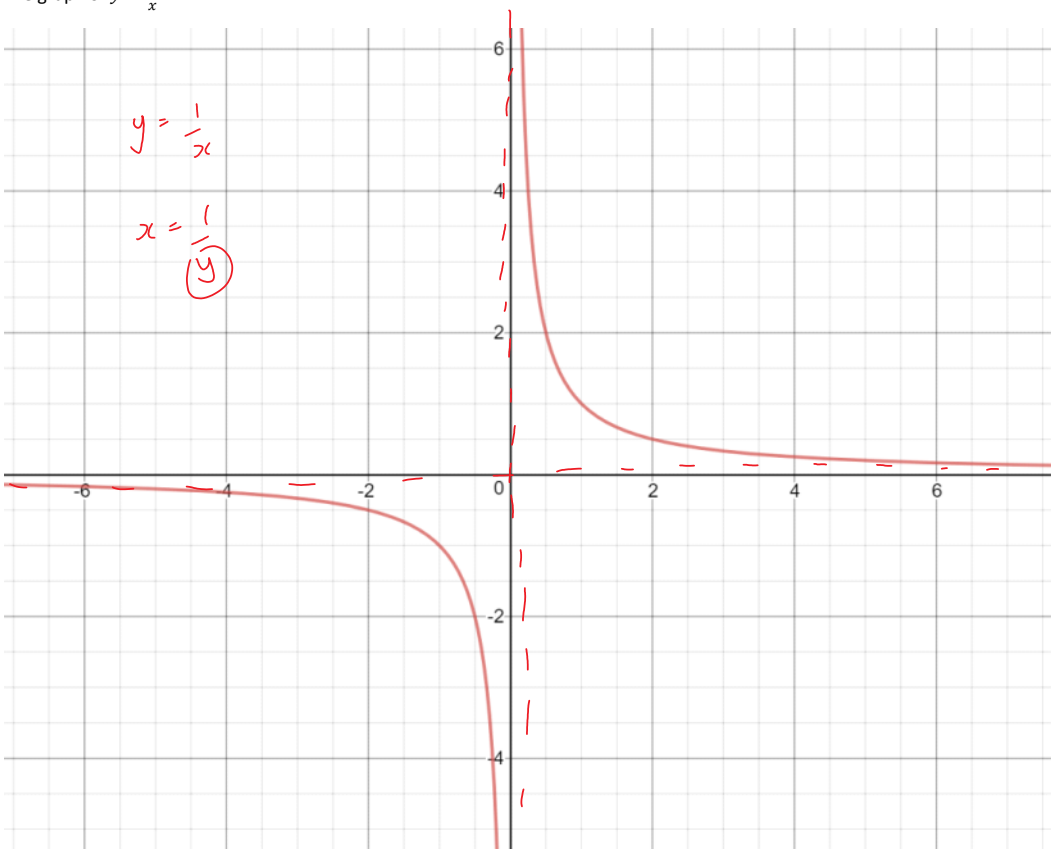
$y = \sqrt{x}$

$x \in [0, \infty)$

$x \in \mathbb{R}^+ \cup \{0\}$

$y \in [0, \infty)$

The graph of $y = \frac{1}{x}$



$y = \frac{1}{x}$

Domain

$x \in \mathbb{R} \setminus \{0\}$

$x \in (-\infty, 0) \cup (0, \infty)$

Range

$y \in \mathbb{R} \setminus \{0\}$

Each of the above graphs can be described in terms of a **domain** and a **range**.
The domains and ranges can be expressed in **interval notation** or other notation.

TERMINOLOGY: MAXIMAL DOMAIN

Barry loves adding other words to make things sound more complex!

What is Maximal Domain?
When the rule for a function is written and the domain isn't given then we give it the largest domain for which the rule makes sense

Writing relations is a tough way

Understanding what the question is trying to tell you is pretty much half the trick!
We can write functions using set notation.
For example:

$$\{(x, y): y = x^2 + 1, x \in \mathbb{R}^+ \cup \{0\}\}$$

Reading this looks tough …. But ask yourself the questions:

- What is the actual equation of the function
- What is the relationship between the letters
- Are they limiting the domain to only certain values?

Examples of how to find domains and ranges

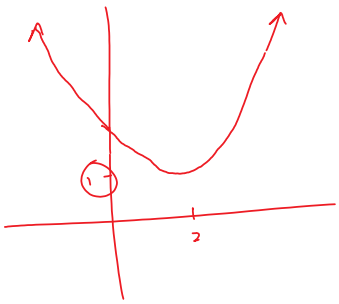
The following examples have been extracted, with permission, from the Cambridge Mathematical Methods Units 1 and 2 Textbook

YOU MUST, MUST, MUST, MUST, MUST DRAW SKETCHES TO HAVE ANY CHANCE OF BEING ABLE TO ANSWER THESE QUESTIONS CORRECTLY.

Example:
For each of the following, complete the square, sketch the graph and state the range. The domain is $\mathbb{R}$.
$y = x^2 - 4x + 5$

$y = x^2 - 4x + 5$

$y = (x-2)^2 + 1$



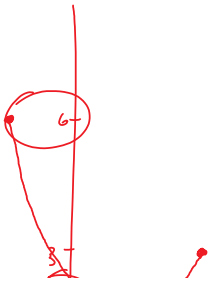
Range $y \in [1, \infty)$

Example:
Sketch the graph of the relation $y = x^2 + 2$ for $x \in [-2,1]$ and state the range.
Note: This is a trick question and appears on exams all the time.
It's testing your understanding and not your ability to do the Mathematics.

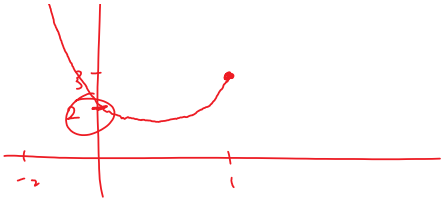
$y = x^2 + 2$

$x = -2 \quad y = 6$

$x = 1 \quad y = 3$



$y = [2, 6]$

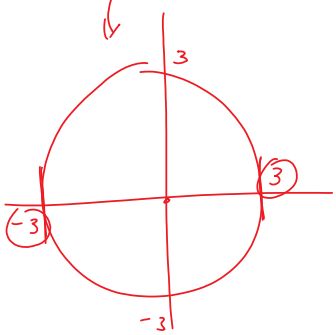


$y = [2, 6]$

Example:
For each of the following relations, state the implied domain and the range:

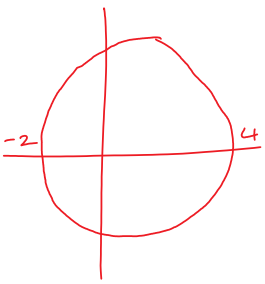
$(x-1)^2 + y^2 = 9$

$x^2 + y^2 = 9$



$x \in [-3, 3]$

$y \in [-3, 3]$



$x \in [-2, 4]$

$y = [-3, 3]$