



Transition matrices: Using recursion

Year 12 General Maths
Units 3 and 4

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Learning Objectives

By the end of the lesson, I would hope that you have an understanding and be able to apply to questions the following concepts:

- To be able to use a matrix recurrence relation $S_0 = \text{initial state matrix}$, $S_{n+1} = TS_n + B$ to model systems that include external additions or reductions at each step of the process.



Recap

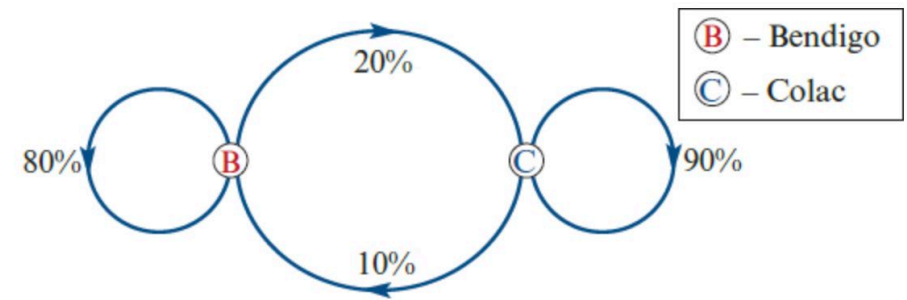
This is the final lesson that you will have to listen to me go on about setting up a car hire company. I promise!

Having set up a car hire business and modelled it using a transition diagram and matrix, I still have the same 90 cars going backwards and forwards.

What if I wanted to keep growing the business? What if demand was outstripping supply? What if I needed more cars?

How could I model this?

$$S_n = \begin{bmatrix} 30 \\ 60 \end{bmatrix} \begin{matrix} B \\ C \end{matrix}$$



Returned to	Rented in	
	Bendigo	Colac
Bendigo	0.8	0.1
Colac	0.2	0.9

$$V_0 = \underline{\hspace{2cm}}, \quad V_{n+1} = R \times V_n \pm B$$



Recap: The rule for finding the state matrix after 'n' steps

In the last lesson we found that we could find the state at any step 'n' by using the following formula. We know T and S_0 .

$$S_n = T^n \times S_0$$

All this really told us was how many cars were going to be in Bendigo and Colac at the end of each week. But I now want to add more more cars in.

$$S_{n+1} = T \times S_n + B$$

Luckily, I want to add the same number of new cars in at the end of each week.

$$B = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

I'm going to add 2 new cars to Bendigo and 2 new cars to Colac each week

We can model this using an updated formula:

$$S_n = T^n \times S_0 + B$$

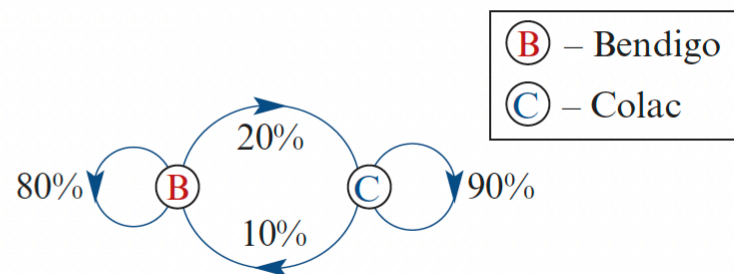
This supposes that we will not remove cars.



Example

A rental starts with 90 cars, 50 located at Bendigo and 40 located at Colac.

Cars are usually rented and returned in the same town. However, a small percentage of cars rented in Bendigo are returned in Colac and vice versa. The transition diagram opposite gives these percentages.



To increase the number of cars, two extra cars are added to the rental fleet at each location each week. The recurrence relation that can be used to model this situation is:

$$S_0 = \begin{bmatrix} 50 \\ 40 \end{bmatrix}, S_{n+1} = TS_n + B \quad \text{where} \quad T = \begin{bmatrix} 0.8 & 0.1 \\ 0.2 & 0.9 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

Determine the number of cars at Bendigo and Colac after:

a 1 week

b 2 weeks.

$$S_1 = T \cdot S_0 + B = \begin{bmatrix} 46 \\ 48 \end{bmatrix}$$

$$S_2 = T \cdot S_1 + B = \begin{bmatrix} 44 \\ 54 \end{bmatrix}$$



Matrix algebra ... to do things backwards

Remember, we can always be asked to do things backwards.

What if I have the formula:

$$S_n = T^n \times S_0 + B$$

But I am given the number of cars in Week 4, and asked to find the number of cars in Week 3?

We would have to use the algebra shown on the right

$$S_{n+1} = T \times S_n + B$$

$$S_{n+1} - B = T \times S_n$$

$$T \times S_n = S_{n+1} - B$$

$$T^{-1} \times T \times S_n = T^{-1}(S_{n+1} - B)$$

$$I \times S_n = T^{-1}(S_{n+1} - B)$$

$$S_n = T^{-1}(S_{n+1} - B)$$

$$S_{n+1} = T^{-1}(S_n - B)$$

Make sure you know this and understand how to use it!



VCAA Questions

The matrix S_{n+1} is determined from the matrix S_n using the recurrence relation $S_{n+1} = T \times S_n - C$, where

$$T = \begin{bmatrix} 0.6 & 0.1 & 0.3 \\ 0.3 & 0.8 & 0.2 \\ 0.1 & 0.1 & 0.5 \end{bmatrix}, \quad S_0 = \begin{bmatrix} 21 \\ 51 \\ 31 \end{bmatrix}, \quad S_1 = \begin{bmatrix} 24.0 \\ 54.3 \\ 20.7 \end{bmatrix}$$

$C = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}$ 1. C
2. S_2

and C is a column matrix.

Matrix S_2 is equal to

A. $\begin{bmatrix} 23.04 \\ 55.78 \\ 16.18 \end{bmatrix}$

B. $\begin{bmatrix} 25.34 \\ 56.28 \\ 17.38 \end{bmatrix}$

C. $\begin{bmatrix} 26.04 \\ 54.78 \\ 18.18 \end{bmatrix}$

D. $\begin{bmatrix} 28.34 \\ 55.28 \\ 19.38 \end{bmatrix}$

E. $\begin{bmatrix} 29.04 \\ 53.78 \\ 20.18 \end{bmatrix}$

$$S_2 = T \cdot S_1 - C$$

$$C = T \times S_n - S_{n+1}$$

$$C = T \cdot S_0 - S_1$$

2021 Further Maths

Paper 1

Question 7

Correct: 40%



VCAA Questions

Further research suggests more shoppers will buy olive oil in the coming months.

A rule to model this situation is $R_{n+1} = T \times R_n + B$, where R_n represents the state matrix describing the number of shoppers n months after July 2021.

$$T = \begin{matrix} & \begin{matrix} \text{this month} \\ C & L & O \end{matrix} \\ \begin{matrix} C \\ L \\ O \end{matrix} & \begin{bmatrix} 0.85 & 0.10 & 0.05 \\ 0.05 & 0.80 & 0.05 \\ 0.10 & 0.10 & 0.90 \end{bmatrix} \end{matrix} \quad \text{next month, } B = \begin{bmatrix} 200 \\ 100 \\ k \end{bmatrix} \begin{matrix} C \\ L \\ O \end{matrix}, \quad R_0 = \begin{bmatrix} 3200 \\ 2000 \\ 2800 \end{bmatrix} \begin{matrix} C \\ L \\ O \end{matrix}$$

k represents the extra number of shoppers expected to buy Ohana olive oil each month.

If $R_2 = \begin{bmatrix} 3333 \\ 2025 \\ 3642 \end{bmatrix}$, what is the value of k ?

$$k = \underline{\underline{200}}$$

$$R_0 = 8000$$

$$R_2 = 9000$$

$$2 \times 1000 = \underline{2000}$$

1 mark

2021 Further Maths

Paper 2

Question 3e

Correct: 15%

$$\begin{matrix} \boxed{\begin{bmatrix} 2 \\ 2 \end{bmatrix}} \\ 90 \quad \boxed{94} \quad \boxed{98} \end{matrix}$$



VCAA Questions

A second market research project also suggested that if the Westmall shopping centre were sold, each of the three centres (Westmall, Grandmall and Eastmall) would continue to have regular shoppers but would attract and lose shoppers on a weekly basis.

Let R_n be the state matrix that shows the expected number of shoppers at each of the three centres n weeks after Westmall is sold.

A matrix recurrence relation that generates values of R_n is

$$R_{n+1} = TR_n + B$$

this week

W	G	E
$\begin{bmatrix} 0.78 & 0.13 & 0.10 \end{bmatrix}$	W	
$\begin{bmatrix} 0.12 & 0.82 & 0.10 \end{bmatrix}$	G	<i>next week,</i>
$\begin{bmatrix} 0.10 & 0.05 & 0.80 \end{bmatrix}$	E	

where $T =$

$$R_1 =$$

$$R_2 = T \cdot R_1 + B$$

$$R_2 - B = T \cdot R_1$$

$$T^{-1}(R_2 - B) = R_1$$

$$B = \begin{bmatrix} -400 \\ 700 \\ 500 \end{bmatrix} \begin{matrix} W \\ G \\ E \end{matrix}$$

The matrix R_2 is the state matrix that shows the expected number of shoppers at each of the three centres in the second week after Westmall is sold.

$$R_2 = \begin{bmatrix} 239\,060 \\ 250\,840 \\ 192\,900 \end{bmatrix} \begin{matrix} W \\ G \\ E \end{matrix}$$

$$241\,000$$

- b. Determine the expected number of shoppers at Westmall in the first week after it is sold.

1 mark

2020 Further Maths
Paper 2
Question 4b
Correct: 20%



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