Differential equations involving a function of the dependent variable

Year 12 Specialist Maths
Units 3 and 4

www.maffsguru.com

Learning Objectives

By the end of the lesson, I hope that you understand and can apply the following to a range of questions from the Unit 3 and 4 Specialist Mathematics course.

- Know how to solve differential equations of the form $\frac{dy}{dx} = g(y)$

Recap of past learning

In the previous lesson we have looked at how to find equations with the independent variable which is pretty much what we have been doing since Year 11.

Now we're going to look at how to solve equations for the dependent variable.

i.e.

$$\frac{dy}{dx} = g(y)$$

This is relatively simple so …

Let's just do some examples!

Find the general solution of each of the following differential equations:

- $\frac{dy}{dx} = 2y + 1, for\ y > -\frac{1}{2}$
- $\frac{dy}{dx} = e^{2y}$
- $\frac{dy}{dx} = \sqrt{1 - y^2},\ for\ y \in (-1,1)$
- $\frac{dy}{dx} = 1 - y^2, for\ -1 < y < 1$

$$\frac{dy}{dx} = 2y + 1$$

$$x = \frac{1}{2}\int \frac{2}{2y+1}\, dy$$

$$x = \frac{1}{2}\ln|2y+1| + c$$

$$x = \frac{1}{2}\ln(2y+1) + c$$

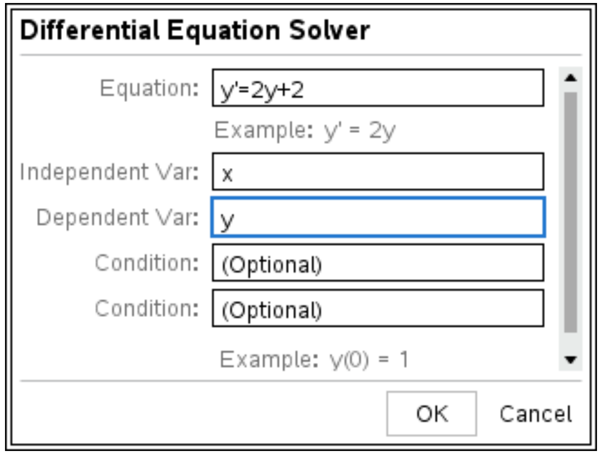
$$2(x-c) = \ln(2y+1)$$

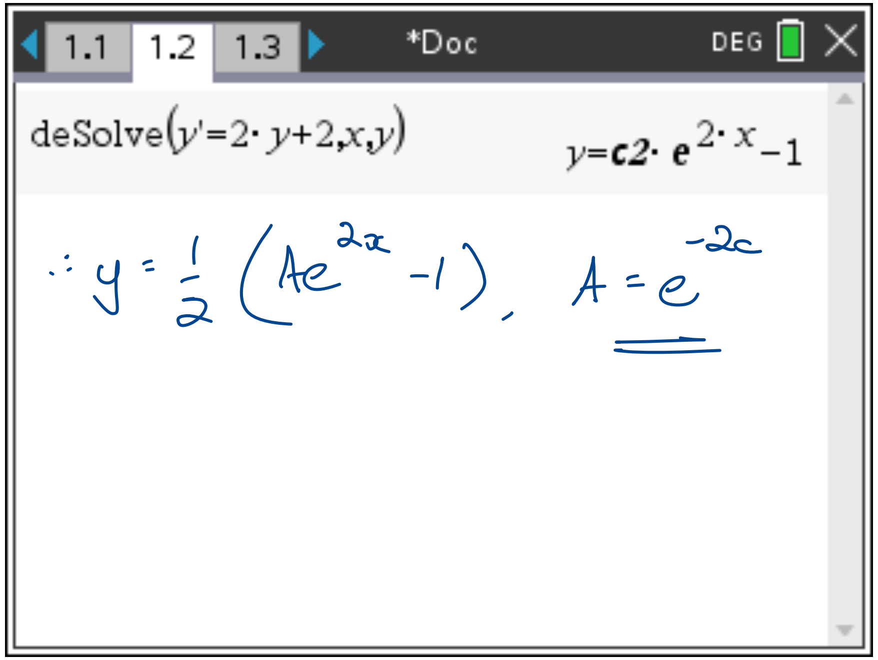
$$e^{2(x-c)} = 2y+1$$

$$2y = e^{2(x-c)} - 1$$

$$y = \frac{1}{2}\left(e^{2(x-c)} - 1\right)$$

$$y = \frac{1}{2}\left(e^{2x} \cdot e^{-2c} - 1\right)$$





$$\therefore y = \frac{1}{2}\left(Ae^{2x} - 1\right), \quad A = e^{-2c}$$

This is relatively simple so …

Let’s just do some examples!

Find the general solution of each of the following differential equations:

- $\frac{dy}{dx} = 2y + 1, for\ y > -\frac{1}{2}$
- $\boldsymbol{\frac{dy}{dx} = e^{2y}}$
- $\frac{dy}{dx} = \sqrt{1 - y^2},\ for\ y \in (-1,1)$
- $\frac{dy}{dx} = 1 - y^2, for\ -1 < y < 1$

$$\frac{dy}{dx} = e^{2y}$$

$$\frac{dx}{dy} = \frac{1}{e^{2y}} = e^{-2y}$$

$$x = \int e^{-2y} . dy$$

$$x = -\frac{1}{2} e^{-2y} + c$$

$$x - c = -\frac{1}{2} e^{-2y}$$

$$e^{-2y} = -2(x - c)$$

$$-2y = \ln(-2(x-c))$$

$$y = -\frac{1}{2} \ln(2c - 2x)$$

$$\underline{\underline{x < c}}$$

This is relatively simple so …

Let's just do some examples!

Find the general solution of each of the following differential equations:

- $\frac{dy}{dx} = 2y + 1, for\ y > -\frac{1}{2}$
- $\frac{dy}{dx} = e^{2y}$
- $\boldsymbol{\frac{dy}{dx} = \sqrt{1 - y^2},\ for\ y \in (-1, 1)}$
- $\boldsymbol{\frac{dy}{dx} = 1 - y^2, for\ -1 < y < 1}$

$$\frac{dy}{dx} = \sqrt{1-y^2}$$

$$\frac{dx}{dy} = \frac{1}{\sqrt{1-y^2}}$$

$$x = \int \frac{1}{\sqrt{1-y^2}} \cdot dy$$

$$x = \sin^{-1}(y) + c$$

$$x - c = \sin^{-1}(y)$$

$$y = \sin(x - c)$$

This is relatively simple so …

Let’s just do some examples!

Find the general solution of each of the following differential equations:

- $\frac{dy}{dx} = 2y + 1, for\ y > -\frac{1}{2}$
- $\frac{dy}{dx} = e^{2y}$
- $\boldsymbol{\frac{dy}{dx} = \sqrt{1 - y^2},\ for\ y \in (-1, 1)}$
- $\boldsymbol{\frac{dy}{dx} = 1 - y^2, for\ -1 < y < 1}$

$$\frac{dy}{dx} = 1 - y^2$$

$$\frac{dx}{dy} = \frac{1}{1-y^2}$$

$$x = \int \frac{1}{1-y^2} \cdot dy$$

$$x = \int \frac{1}{(1-y)(1+y)} \cdot dy$$

$$x = -\frac{1}{2}\int \frac{1}{1-y} \cdot dy + \frac{1}{2}\int \frac{1}{1+y} \cdot dy$$

$$x = -\frac{1}{2} \ln(1-y) + \frac{1}{2} \ln(1+y) + c$$

$$2(x - c) = \ln(1+y) - \ln(1-y)$$

$$2(x-c) = \ln\left(\frac{1+y}{1-y}\right)$$

$$e^{2x-2c} = \frac{1+y}{1-y}$$

$$e^{2x-2c}(1-y) = (1+y)$$

$$y = \frac{Ae^{2x} - 1}{Ae^{2x} + 1}$$

$$\underline{\underline{A = e^{-2c}}}$$

Learning Objectives: Reviewed

By the end of the lesson, I hope that you understand and can apply the following to a range of questions from the Unit 3 and 4 Specialist Mathematics course.

- Know how to solve differential equations of the form $\frac{dy}{dx} = g(y)$

Questions to complete

The following represents an indication of the minimum number of questions to complete for this exercise. If you choose to do more, then all good. Note that you should also aim to complete some questions from Chapter Reviews too.

Chapter 11C : Differential equations involving a function of the dependent variable
Questions: 1acegi, 2bdfh, 3