Discrete Random Variables

Year 11
Mathematical Methods

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Learning Objectives

By the end of the lesson, I hope that you understand and can apply the following to a range of questions from the Year 11 Mathematical Methods course.

- Understand what it means to sample without replacement
- Understand the effect this has on a probability distribution.
- Understand what it means to be a hypergeometric distribution

RECAP

This very short chapter is dealing with discrete probability distributions. In the previous section, we looked at discrete random distributions. With the examples we looked at it seemed the objects were always replaced or reused.

But what happens if we don't replace when we sample?

Taking your hand out of the sweetie jar

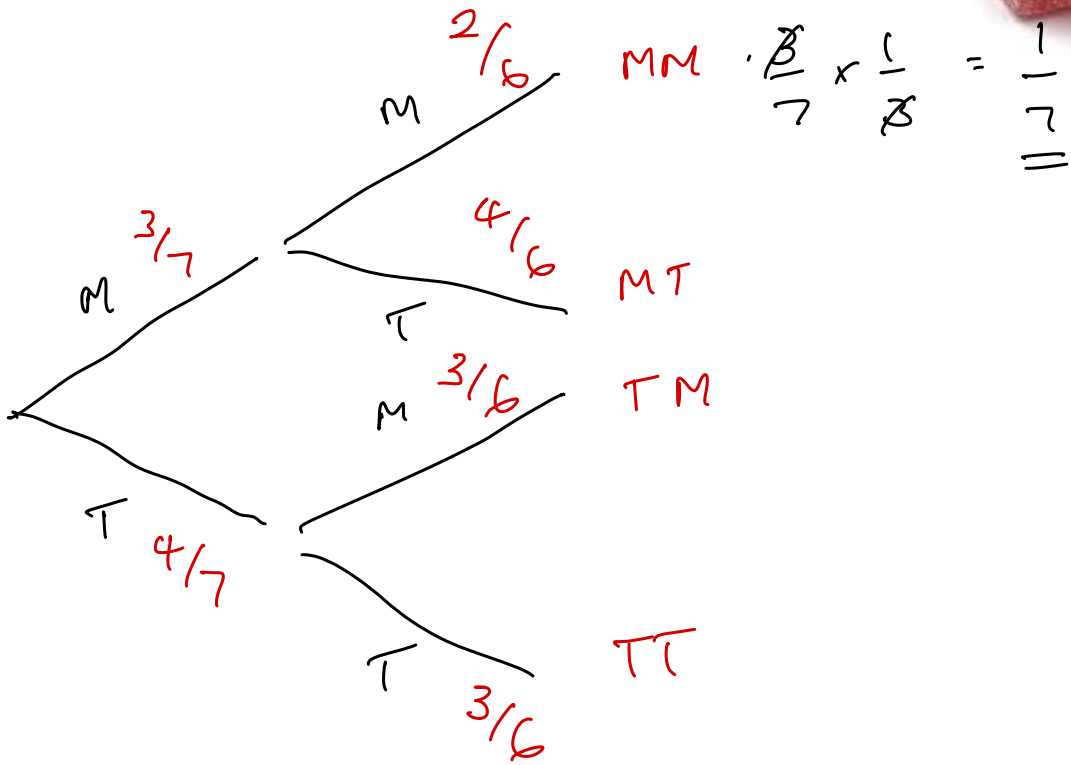
I have a jar containing not cola cubes … but three mints and four toffees.

I am going to select two sweets (without looking) and I want to see how many mints I am going to get. We will call this the random variable *X*.

It is clear that this number is either going to be 0, 1 or 2.

I am not going to replace the sweet once it is taken.

What will this do to my probabilities?



$MM \cdot \frac{3}{7} \times \frac{1}{3} = \frac{1}{7}$

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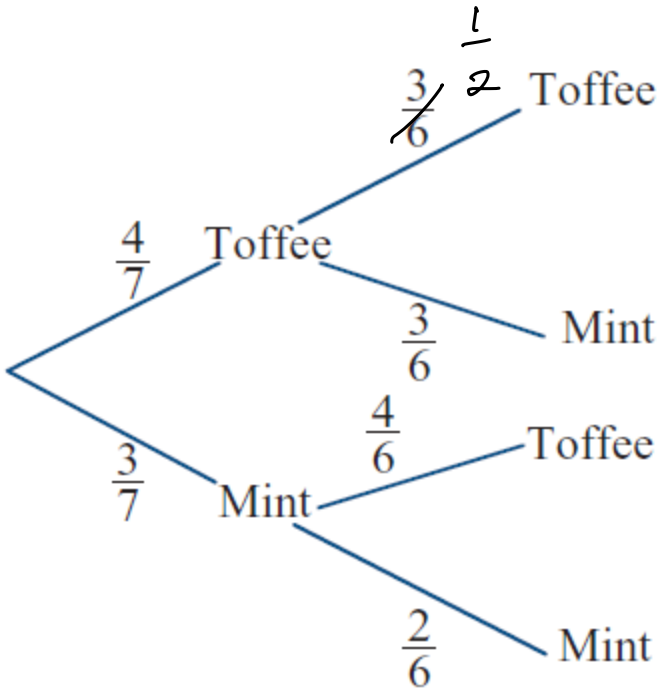

$\frac{4}{7} \times \frac{3}{6} = \frac{12}{42}$

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Probability Distribution for the number of mints

We can now put this in a table as shown below:

x	0	1	2
$\Pr(X = x)$	$\frac{2}{7}$	$\frac{4}{7}$	$\frac{1}{7}$

Question: What happens if we fill a massive jar of sweets with lots of mints and toffees. It wouldn't be practical to draw a tree diagram to find the probabilities? So, there must be another way to get the values above.

Another way of finding the probabilities …

Let's firstly think of getting zero mints. As the order they are taken isn't important, we can find the combinations of individual selections of mints and toffees and multiply them together.

x	0	1	2
$\Pr(X = x)$	$\frac{2}{7}$	$\frac{4}{7}$	$\frac{1}{7}$

✓ ✓

Number of ways of getting 2 toffees AND 0 mints
= 4C2 and 3C0.

Number of ways of getting 1 toffee AND 1 mints
= 4C1 and 3C1

Number of ways of getting 0 toffees AND 2 mints
= 4C0 and 3C2.

Total number of ways of selecting 2 from 7
= 7C2

↑ 21

T M

$= 6/21 = 2/7$ $\quad {}^4C_2 \quad {}^3C_0 = 6 \times 1 = 6$

$= 12/21 = 4/7$

T T

$= 3/21 = 1/7$ $\quad {}^4C_1 \quad {}^3C_1 = 4 \times 3 = 12$

T M

${}^4C_0 \quad {}^3C_2 = 1 \times 3$

Hypergeometric distribution: Come again?

When we look at a distribution where we sample without replacement, we call it a hypergeometric distribution.

Example 1

Marine biologists are studying a group of dolphins which live in a small bay. They know there are 12 dolphins in the group, four of which have been caught, tagged and released to mix back into the population. If the researchers return the following week and catch another group of three dolphins, what is the probability that two of these will already be tagged?

$$\frac{{}^{T}4C_{2} \times {}^{N}8C_{1}}{{}^{12}C_{3}} = \frac{12}{55}$$

Learning Objectives: Revisited

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Questions to complete

The following are the minimum number of questions you are expected to answer. There is nothing wrong with answering more!

Ex 11B
Questions: TBA

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