Transition matrices: Setting up a transition matrix

Year 12 General Maths
Units 3 and 4

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Learning Objectives

By the end of the lesson, I would hope that you have an understanding and be able to apply to questions the following concepts:

- To be able to set up a transition matrix from a diagram or written information.

Recap

This is the first lesson in this section of the course. We have looked, in the previous section at the basics of matrices. We are now going to need to build on the ideas and apply them to real world concepts/

As I have said many times before, this section requires you to have a deep understanding of why you're doing this and what a matrix stands for. You aren't going to be able to "just bang it into a CAS".

I’m leaving teaching!

Yup. I’ve decided that I’m going to start my own hire company. I‘m not going to hire myself out .. as who is going to pay for that … but I am going to hire out cars.

Seems like a great idea.

Melbourne is saturated and so where to set up shop?

What two better places to start a car hire company than Bendigo and Colac.

I've never been there but the photos make them both look nice.

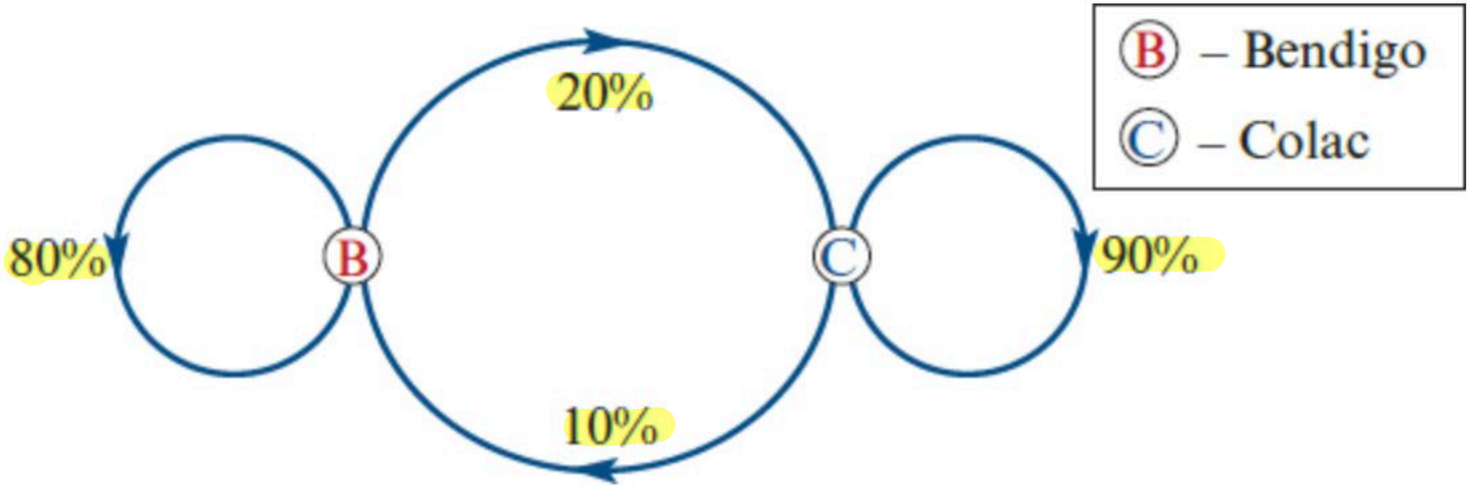
Cars don’t always come back to where you hire them from

I learned, on a trip to the UK, that it’s perfectly acceptable to return a car to a different place than where you hired it from.

So, I hired a car in Cambridge and left it at Heathrow Airport.

However, mostly, people return cars to where they hire them from.

We can express this in a pretty picture.

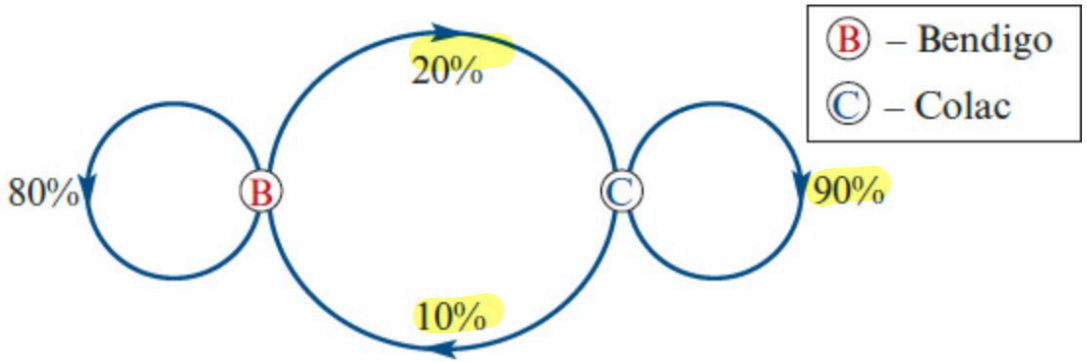


From week to week:

- 0.8 (or 80%) of cars rented each week in Bendigo are returned to Bendigo
- 0.2 (or 20%) of cars rented each week in Bendigo are returned to Colac
- 0.1 (or 10%) of cars rented each week in Colac are returned to Bendigo
- 0.9 (or 90%) of cars rented each week in Colac are returned to Colac.

But what about the matrix?

This transition diagram can also be expressed as a **transition matrix.**



$$\begin{array}{l} \\ \text{Returned to} \end{array} \begin{array}{c} \begin{array}{cc} \textit{Bendigo} & \textit{Colac} \end{array} \\ \begin{bmatrix} 0.8 & 0.1 \\ 0.2 & 0.9 \end{bmatrix} \end{array}$$

Rented in

Note: The order of the "start" and "finish" is now different.

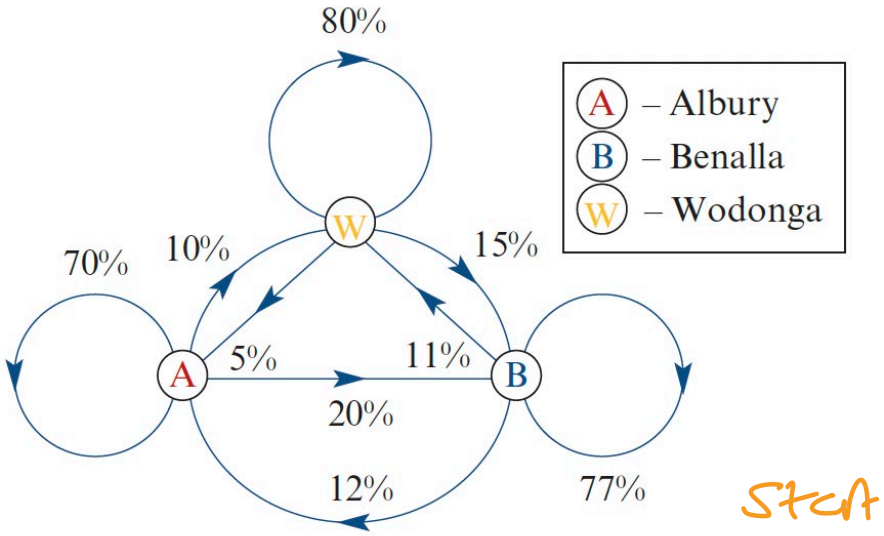
This is really important to note and to remember when you do this for exam questions.

Example

The business didn't go too well in Bendigo. Not too many people wanted to use my cars.

So, I'm going to try and set up again. This time in three places.

For fun, let's set up a transition matrix to describe the transition diagram shown!



start

end \ start	A	B	W
A	0.7	0.12	0.05
B	0.2	0.77	0.15
W	0.1	0.11	0.8

Note: The numbers which are placed in the matrix are always decimal representations of percentages i.e. a number between 0 and 1.

Note: The columns of a transition matrix always add to one.

Example 2: Setting up another transition matrix

A factory has a large number of machines. Machines can be in one of two states: operating or broken. Broken machines are repaired and come back into operation, and vice versa. On a given day:

- 85% of machines that are operational stay operating
- 15% of machines that are operating break down
- 5% of machines that are broken are repaired and start operating again
- 95% of machines that are broken stay broken.

Construct a transition matrix to describe this situation. Use the columns to define the situation at the 'Start' of the day and the rows to describe the situation at the 'End' of the day.

Start

End	O	B
O	0.85	0.05
B	0.15	0.95

VCAA Questions

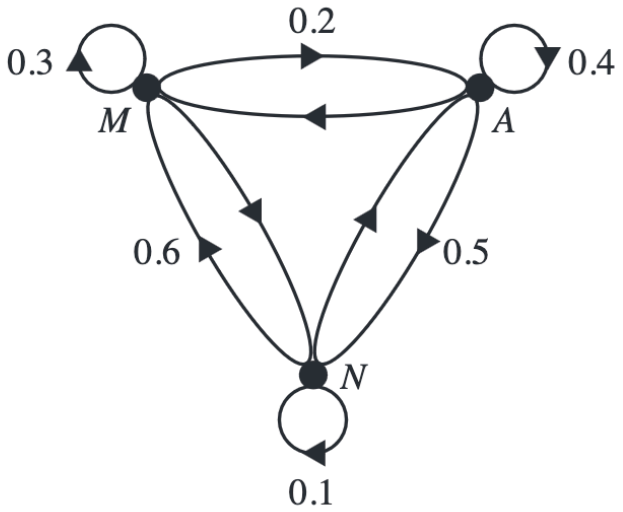
2022 Further Maths
Paper 1
Question 3
Correct: 71%

Question 3

Each day, members of a swim centre can choose to attend a morning session (M), an afternoon session (A) or no session (N).

The transition diagram below shows the transition from day to day.

The transition diagram is incomplete.



Which one of the following transition matrices represents this transition diagram?

A.

	this day: M	A	N	
	0.2	0.1	0.1	M
	0.3	0.5	0.3	A
	0.5	0.4	0.6	N

next day

B.

	this day: M	A	N	
	0.3	0.1	0.6	M
	0.5	0.4	0.3	A
	0.2	0.5	0.1	N

next day

C.

	this day: M	A	N	
	0.3	0.1	0.6	M
	0.2	0.4	0.3	A
	0.5	0.5	0.1	N

next day

D.

	this day: M	A	N	
	0.3	0.5	0.2	M
	0.1	0.4	0.7	A
	0.6	0.1	0.1	N

next day

E.

	this day: M	A	N	
	0.3	0	0.6	M
	0.2	0.4	0	A
	0	0.5	0.1	N

next day

VCAA Questions

2021 Further Maths
Paper 1
Question 2
Correct: 73%

Every Friday, the same number of workers from a large office building regularly purchase their lunch from one of two locations: the deli, D, or the cafe, C.

It has been found that:

- of the workers who purchase lunch from the deli on one Friday, 65% will return to purchase from the deli on the next Friday
- of the workers who purchase lunch from the cafe on one Friday, 55% will return to purchase from the cafe on the next Friday.

A transition matrix that can be used to describe this situation is

A. *this Friday*

	D	C	
	0.55	0.35	D
	0.45	0.65	C

next Friday

B. *this Friday*

	D	C	
	0.65	0.45	D
	0.45	0.55	C

next Friday

C. *this Friday*

	D	C	
	0.65	0.55	D
	0.45	0.55	C

next Friday

D. *this Friday*

	D	C	
	0.65	0.45	D
	0.35	0.55	C

next Friday

E. *this Friday*

	D	C	
	0.65	0.55	D
	0.35	0.45	C

next Friday

this.

next	D	C
D	0.65	0.45
C	0.35	0.55

VCAA Questions

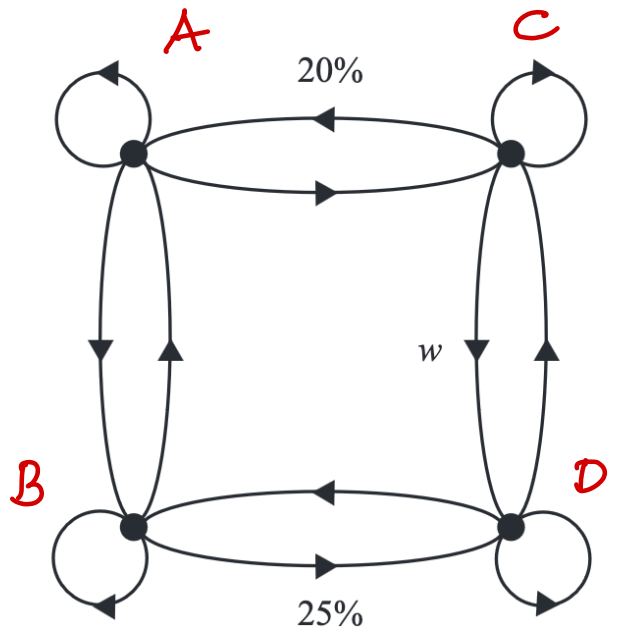
2021 Further Maths
Paper 1
Question 6
Correct: 66%

A fitness centre offers four different exercise classes: aerobics (A), boxfit (B), cardio (C) and dance (D).

A customer's choice of fitness class is expected to change from week to week according to the transition matrix P, shown below.

$$P = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0.65 & 0 & 0.20 & 0.10 \\ 0 & 0.65 & 0.10 & 0.30 \\ 0.20 & 0.10 & 0.70 & 0 \\ 0.15 & 0.25 & 0 & 0.60 \end{bmatrix} \end{array} \begin{array}{l} \text{this week} \\ \\ \\ \\ \text{next week} \end{array}$$

An equivalent transition diagram has been constructed below, but the labelling is not complete.



15%

The proportion for one of the transitions is labelled w.

The value of w is

A. 10%
B. 15%
C. 20%
D. 25%
E. 30%

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