Discrete Random Variables

Year 11

Mathematical Methods

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Learning Objectives

By the end of the lesson, I hope that you understand and can apply the following to a range of questions from the Year 11 Mathematical Methods course.

- Understand what it means to be a random variable
- Understand what it means to be a discrete probability distribution
- Know how to apply this to a range of questions

RECAP

Probability isn't anything new in Mathematics. We have been finding the probabilities of things since Year 6. Now it's time to look at things in a slightly different way by looking at probability distributions and, in particular, discrete probability distributions.

Before we do that, let's look at what it means to be a random variable.

Random Variables

When we toss a coin, we know that the outcome from that one toss will either be a Head of a Tail. We know that the outcome will be random (unless we are using a biased coin). There will be no way to predict the outcome.

What if we were to throw the same coin three times and look at counting only how many heads were observed.

The results from one toss will be random. All outcomes will be random. So, in this instance we will say that the number of heads gained will be a random variable. We can assign this a letter which is normally X.

Sample space for throwing a Head

So, back to the example we had. We are throwing it three times, and looking at how many times a head might be obtained. Here, order is important.

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$$\epsilon = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$$

HHH

HHT

HTH

$\vdots$

TTT

Outcome	Number of heads
HHH	$X = 3$
HHT	$X = 2$
HTH	$X = 2$
THH	$X = 2$
HTT	$X = 1$
THT	$X = 1$
TTH	$X = 1$
TTT	$X = 0$

We can tabulate the information as shown above. Remember, X is the number of heads obtained in three throws of a coin.

Random variables

We can see that the possible values of X are 0, 1, 2 and 3.

A **random variable** is a function which assigns a number to each outcome in the sample space.

A random variable can be discrete or continuous.

A discrete random variable is one which can only take a countable number of distinct values.

A continuous random variable is one which can take any value in an interval of the real numbers and is usually generated by measuring.

Outcome	Number of heads
HHH	$X = 3$
HHT	$X = 2$
HTH	$X = 2$
THH	$X = 2$
HTT	$X = 1$
THT	$X = 1$
TTH	$X = 1$
TTT	$X = 0$

Discrete Probability Distributions

We can determine the probability of each value of the random variable occurring.

Outcome	Number of heads
HHH	$X = 3$
HHT	$X = 2$
HTH	$X = 2$
THH	$X = 2$
HTT	$X = 1$
THT	$X = 1$
TTH	$X = 1$
TTT	$X = 0$

$\frac{1}{8}$

$\frac{3}{8}$

$\frac{3}{8}$

$\frac{1}{8}$

Discrete Probability Distributions

And can write these probabilities in a table of results

x	0	1	2	3
$\Pr(X = x)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

This notation is really important to understand. It means the probability of getting a particular outcome for the random variable *X*. In this case the probability of getting 0, 1, 2 or 3 heads when rolling a die three times.

Formal definition

The following is really, really important and is used throughout this year and next!

The **probability distribution** of a discrete random variable X is a function

$$p(x) = \Pr(X = x)$$

that assigns a probability to each value of X. It can be represented by a rule, a table or a graph, and must give a probability $p(x)$ for every value x that X can take.

For any discrete probability distribution, the following two conditions must hold:

1. Each value of $p(x)$ belongs to the interval $[0, 1]$. That is, $0 \le p(x) \le 1$ for all x.
2. The sum of all the values of $p(x)$ is 1.

x	0	1	2	3
$\Pr(X = x)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

As these are probabilities, individual values should not be outside the range of 0 to 1.

Note that the probabilities all add to 1.

Example 1

Consider the probability distribution:

x	1	2	3	4	5	6
$\Pr(X = x)$	0.2	0.3	0.1	0.2	0.15	0.05

Use the table to find:

a $\Pr(X = 3)$ 0.1 **b** $\Pr(X < 3)$ 0.5 **c** $\Pr(X \geq 4)$ 0.4

d $\Pr(3 \leq X \leq 5)$ 0.45 **e** $\Pr(X \neq 5)$ 0.85

$\Pr(X = x)$

$\uparrow$

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$\Pr(X = 1) + \Pr(X = 2)$

Example 2

Consider the function:

x	1	2	3	4	5
$\Pr(X = x)$	$2c$	$3c$	$4c$	$5c$	$6c$

For what value of c is this a probability distribution?

$2c + 3c + 4c + 5c + 6c = 1$

$20c = 1$

$c = \frac{1}{20}$

Example 3

The table shows a probability distribution for a random variable X.

x	1	2	3	4	5	6
$\Pr(X = x)$	0.2	0.2	0.07	0.17	0.13	0.23

Give the following probabilities:

a $\Pr(X > 4)$

0.36

b $\Pr(2 < X < 5)$

0.24

c $\Pr(X \geq 5 \mid X \geq 3)$

$$= \frac{\Pr(X \geq 5 \cap X \geq 3)}{\Pr(X \geq 3)}$$

$$= \frac{0.36}{0.6}$$

$$= 0.6$$

Example 4

The following distribution table gives the probabilities for the number of people on a carnival ride at a particular time of day.

Number of people (t)	0	1	2	3	4	5
$\Pr(T = t)$	0.05	0.2	0.3	0.2	0.1	0.15

Find:

a $\Pr(T > 4)$ 0.15

b $\Pr(1 < T < 5)$ 0.6

c $\Pr(T < 3 \mid T < 4)$

$$= \frac{\Pr(T<3 \cap T<4)}{\Pr(T<4)}$$

$$= \frac{\Pr(T<3)}{\Pr(T<4)}$$

$$= \frac{0.55}{0.75}$$

$$= 0.7\dot{3}$$

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Questions to complete

The following are the minimum number of questions you are expected to answer. There is nothing wrong with answering more!

Ex 11A
Questions: ALL

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