The equation of a straight line

Wednesday, 16 January 2019 8:33 am

By the end of the lesson I would hope that you have an understanding of the concepts below which you can apply to a number of complex questions:

- Know the many forms we can write an equation of a straight line
  - Gradient intercept form
  - Intercept form
  - **Point gradient form**
- Know how to find the equation of a line given certain conditions

RECAP

I started this course with a lesson saying how hard Methods 1 and 2 is!
It really doesn't seem like it from the work which is being covered in this chapter!
Much of it seems like a review of the work done in previous years.

And ... to be honest ... it is.
Much of the early work in this series is designed to make sure your algebra skills are top notch.
That you can take worded questions, extract the important information and then apply the correct Mathematics to it.

Gradient-Intercept form

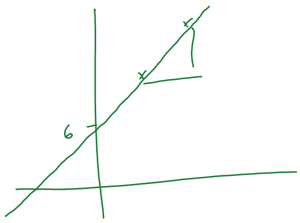
You've been working with this since Year 9.

$$y = mx + c$$

$y = mx + c$

We can read information from the equation and identify gradients and intercepts, or use the same information to find the equation of the straight line.

$y = 3x + 6$



Intercept-form

You have already met in the intercept form in a previous topic

$$3x + 4y = 7$$
$$2x - 3y = -4$$

We love intercept-form!!!

$$3x + 6y = 12$$

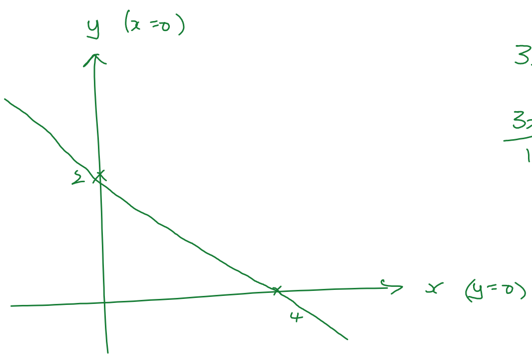
$x = 0$ $\quad 6y = 12 \quad (0,2)$

$y = \underline{\underline{2}}$

$y = 0$ $\quad 3x + \cancel{6y} = 12 \quad (4,0)$

$3x = 12$

$x = \underline{\underline{4}}$



$3x + 6y = 12$

$\frac{3x}{12} + \frac{6y}{12} = 1$

$\frac{x}{4} + \frac{y}{2} = 1$

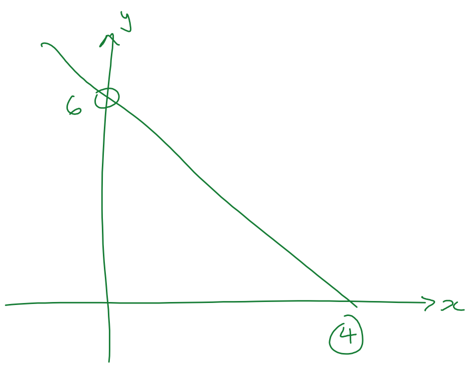
Let's throw in a formula!

$$\frac{x}{a} + \frac{y}{b} = 1$$

$\frac{x}{4} + \frac{y}{6} = 1$

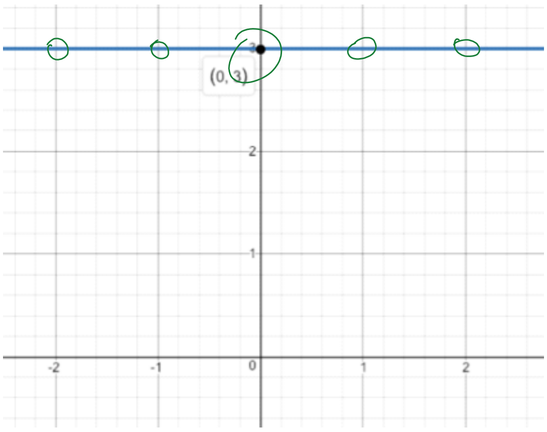
$\times 4$ $\quad x + \frac{4y}{6} = 4$

$6x + 4\underline{\underline{y}} = 24$



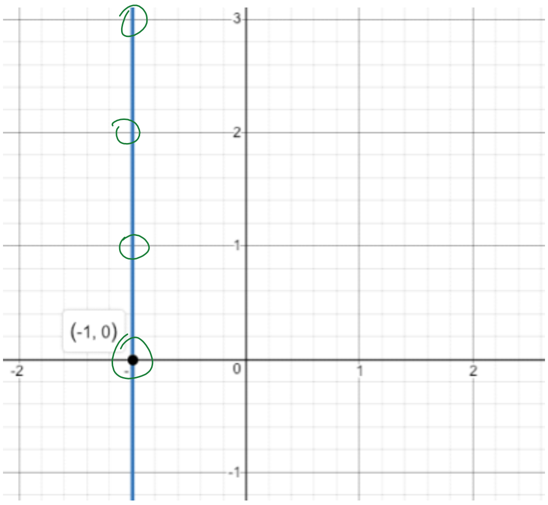
Special Lines

Horizontal Lines



$y = 3$

Vertical lines



$x = -1$

Point-gradient form

This is my personal favourite!
It looks funky … but it's built from the equations above.

$$\boxed{y - y_1 = m(x - x_1)}$$

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$m = \dfrac{y - y_1}{x - x_1}$

$y - y_1 = m(x - x_1)$

Example:
Taken from the Cambridge Essentials Textbook Series

Find the equation of the line which passes through the point $(-1,3)$ and has gradient 4.

$y - y_1 = m(x - x_1)$

$y - 3 = 4(x - (-1))$

$y - 3 = 4(x + 1)$

$y - \cancel{3} = 4x + 4$

$y = 4x + 7$

Example:
Taken from the Cambridge Essentials Textbook Series

Find the equation of the straight line passing through the points $(1,-2)$ and $(3,2)$.

$x_1 \; y_1$ $(1, -2)$ $x_2 \; y_2$ $(3, 2)$

$y - y_1 = m(x - x_1)$

$y - 2 = 2(x - 3)$

$y - \cancel{2} = 2x - 6$

$y = 2x - 4$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{2 - (-2)}{3 - 1}$

$= \frac{4}{2}$

$= 2$

General form of the equation of a straight line

This is one to throw into the mix.
We have seen that equations of straight lines can be written in a number of ways:

$y = mx + c$

$y = 3x + 2$
$3x - y = -2$
$3x - y + 2 = 0$

$y = 3x + 2$

$3x - y = -2$

$3x - y + 2 = 0$

We can hence say that the general form of the equation of a straight line is:

$mx + ny + p = 0$

$mx + ny + p = 0$