

Simultaneous linear equations

Thursday, 17 January 2019

5:07 PM

★ By the end of the lesson I would hope that you have an understanding of the concepts below which you can apply to a number of complex questions:

- Understand that two lines can have zero, one or infinite solutions.
- Use the fact that two lines can have zero, one or infinite solutions to solve equations and find unknowns.
- Use the CAS to help solve simultaneous equations

RECAP:

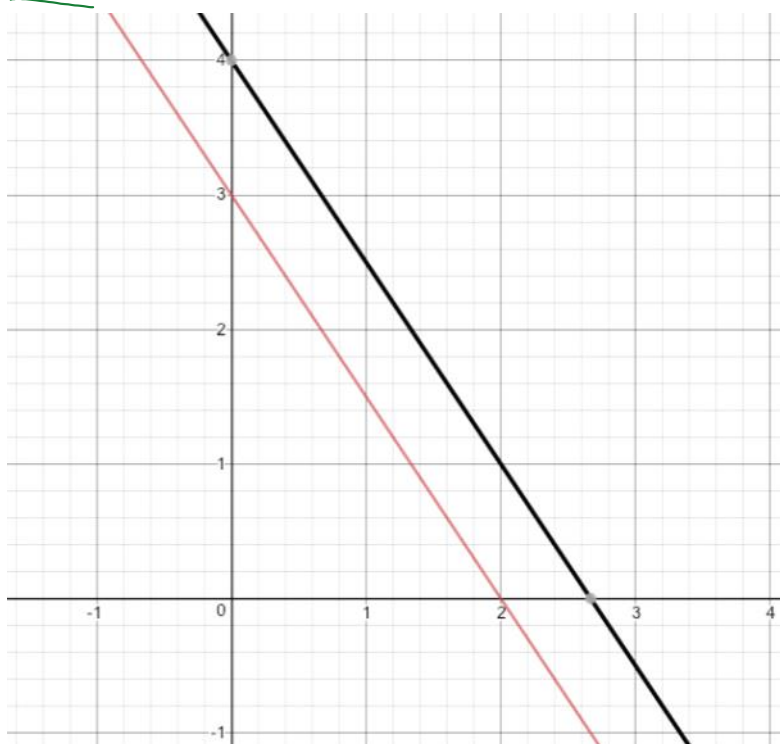
I really do feel like I've taught this lesson 100 times!

This one topic seems to have been in Year 10, 11 and 12 Mathematics so you must conclude that it's really important!

It is important to know that, when we have two lines they can meet in zero, one or infinitely many places.

Consider the following simultaneous equations:

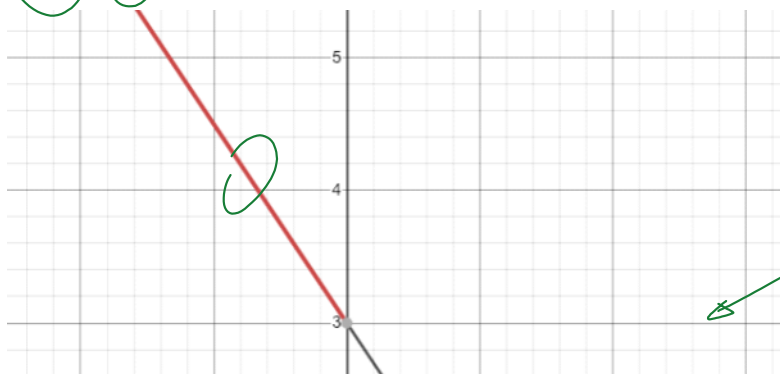
$$\begin{cases} 3x + 2y = 6 \\ 3x + 2y = 8 \end{cases}$$



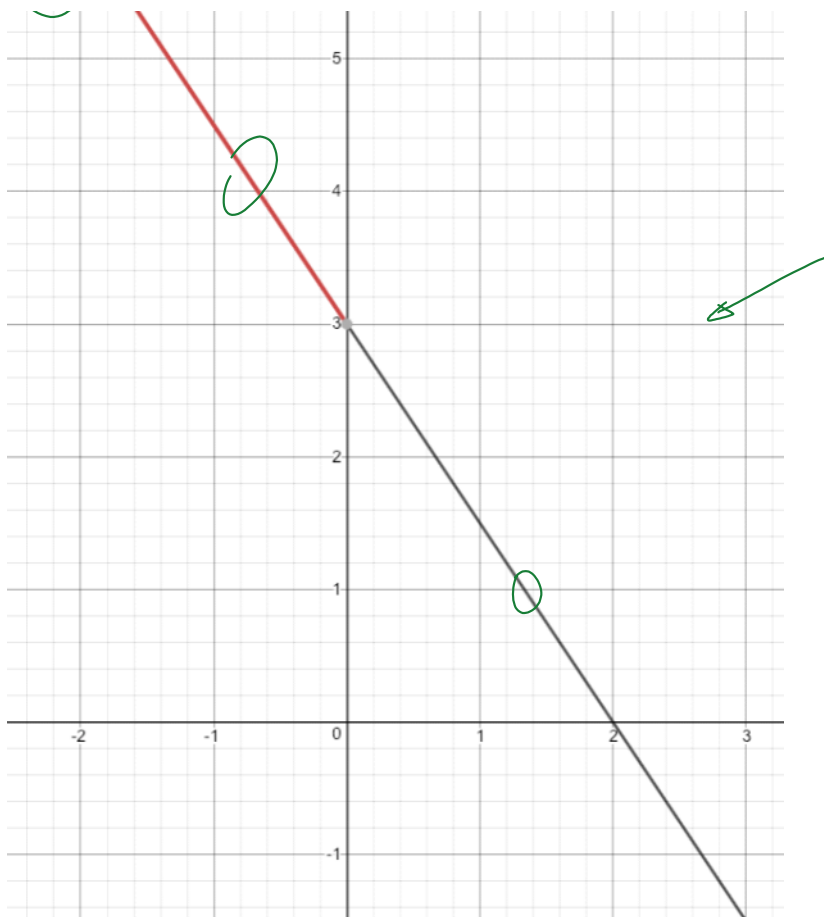
These lines are parallel and hence will never meet.
There are no solutions to this equation.

Consider the following simultaneous equations:

$$\begin{cases} 3x + 2y = 6 \\ 6x + 4y = 12 \end{cases}$$



$$\begin{aligned} 12 \swarrow & \begin{cases} 3x + 2y = 6 \\ 6x + 4y = 12 \end{cases} \end{aligned}$$

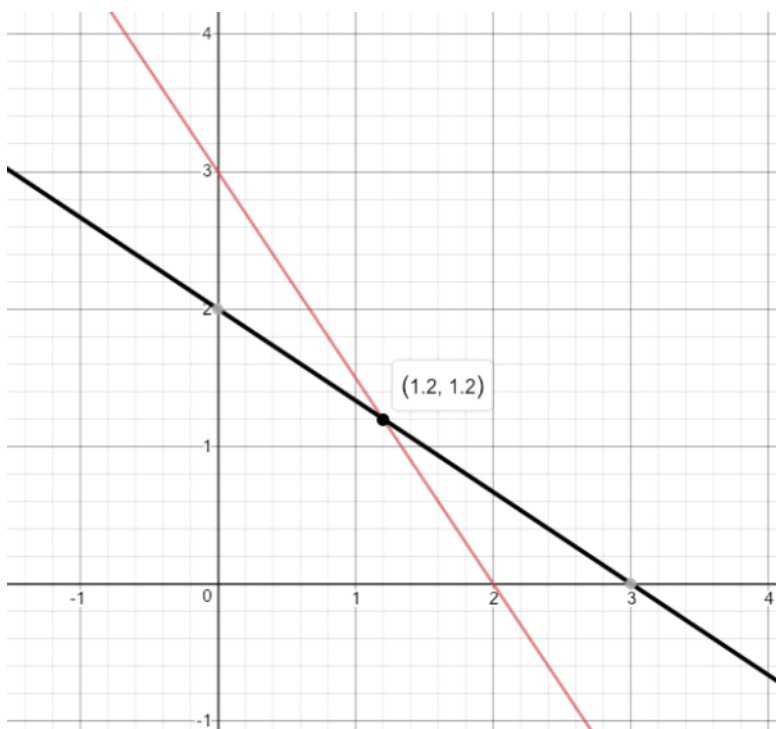


$$\begin{aligned} 3x + 2y &= 6 \\ 6x + 4y &= 12 \end{aligned}$$

These lines are the same and hence there are infinitely many solutions.
We need to use a parameter to describe the solutions to this.

Consider the following simultaneous equations:

$$\begin{cases} 3x + 2y = 6 \\ 2x + 3y = 6 \end{cases}$$



These lines meet in one place.
Hence they have one solution (which should be expressed as a coordinate)

Examples of how we use the above information

Example 1:

The simultaneous equations $2x + 3y = 6$ and $4x + 6y = 12$ have infinitely many solutions. Describe these solutions through the use of a parameter.

$$\begin{array}{l} \times 2 \quad \left. \begin{array}{l} 2x + 3y = 6 \\ 4x + 6y = 12 \end{array} \right\} \\ \lambda = 1 \quad \left(\frac{3}{2}, 1 \right) \end{array} \quad \begin{array}{l} 2x + 3y = 6 \\ 2x = -3y + 6 \\ x = -\frac{3y+6}{2} \\ y = \lambda \\ x = -\frac{3\lambda+6}{2} \end{array} \quad \left(-\frac{3\lambda+6}{2}, \lambda \right)$$

Note: This can also be done on a CAS

Example 2:

The family of lines $y = mx + 2$ with varying gradient m all pass through the point $(0, 2)$.

- For what values of m does the line $y = mx + 2$ not intersect the line $y = 5x - 3$?
- For what values of m does the line $y = mx + 2$ intersect the line $y = 5x - 3$?
- If the line $y = mx + 2$ intersects the line $y = 5x - 3$ at the point $(5, 22)$, find the value of m .

$$\begin{array}{l} y = mx + 2 \quad (0, 2) \quad x = 0 \quad y = 2 \\ \bullet \quad m = 5 \\ \bullet \quad m \neq 5 \\ \bullet \quad \boxed{y = mx + 2} \quad y = 5x - 3 \quad (5, 22) \\ 22 = m(5) + 2 \\ 20 = 5m \\ m = 4 \end{array}$$

Example 3:

The lines $(m - 2)x + y = 2$ and $mx + 2y = k$ intersect at $(2, 8)$. Find the values of m and k .

$$\begin{array}{l} (m-2)x + y = 2 \\ x=2 \quad (m-2)(2) + 8 = 2 \\ y=8 \quad 2m - 4 + 8 = 2 \\ \quad 2m + 4 = 2 \\ \quad 2m = -2 \\ \quad m = -1 \end{array} \quad \begin{array}{l} mx + 2y = k \\ (-1)(2) + 2(8) = k \\ -2 + 16 = k \\ k = 14 \end{array}$$

$$m = -1$$

Example 4:

Using other theory - another approach

Consider the simultaneous linear equations $(m-2)x + y = 2$ and $mx + 2y = k$. Find the values of m and k such that the system of equations has:

- no solution
- infinitely many solutions
- a unique solution.

$$\begin{bmatrix} m-2 & 1 \\ m & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ k \end{bmatrix}$$

$$\begin{aligned} (m-2)x + y &= 2 \\ mx + 2y &= k \end{aligned}$$

$$(m-2)2 - m = 0$$

$$2m - 4 - m = 0$$

$$m - 4 = 0$$

$$\boxed{m = 4}$$

Unique soln $m \neq 4$
 $k \in \mathbb{R}$

$$\begin{aligned} (4-2)x + y &= 2 \\ 4x + 2y &= k \end{aligned}$$

$$m = 4 \quad k \neq 4$$

$$\begin{array}{l} \times 2 \\ \hline \begin{aligned} 2x + y &= 2 \\ 4x + 2y &= k \end{aligned} \\ \hline \begin{aligned} 4x + 2y &= 4 \\ 4x + 2y &= k \end{aligned} \\ \hline \end{array}$$

$\therefore$ Zero solns $m = 4 \quad k \neq 4$

$\therefore$ ∞ solns $m = 4 \quad k = 4$