Simultaneous equations

Saturday, 12 January 2019 11:08 am

By the end of the lesson I would hope that you have an understanding of the concepts below which you can apply to a number of complex questions:

- Know what a simultaneous equation is
- Know how to solve them using the following methods:
  - Substitution
  - Elimination
  - CAS
- Understand the geometry of simultaneous equations

RECAP:

If you're really lucky, you had a teacher back in Year 9 or 10 who told you that simultaneous equations were simply two straight lines which might **meet** at one point. When you solve simultaneous equations you are effectively finding where the two lines meet. The solution is a co-ordinate.

You would have been shown how to solve them using two main methods:

- Substitution, and
- Elimination

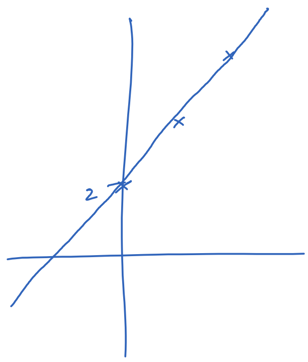
If you were really unlucky you had to solve them graphically! By hand ...

The geometry of simultaneous equations

In Year 9 or 10 we only told you part of the story.
Straight lines don't always meet!
But, before we go further ...

$y = 3x + 2$

$y = mx + c$



Two ways to express a straight line:

Gradient/intercept form: $y = 3x + 2$

Intercept form: $3x - 2y = 6$

Note: The intercept form is our preferred way to solve by elimination.

Compare the following two straight lines:

$3x - 2y = 6$
$6x - 4y = 12$

$\times 2$ $\quad 3x - 2y = 6$

$6x - 4y = 12$

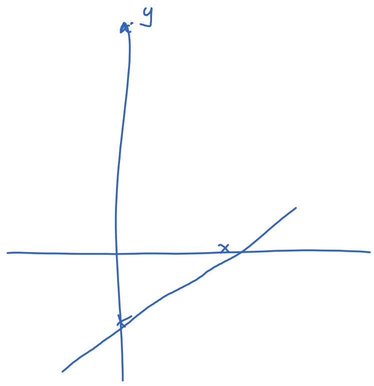
What do you notice?

$x = 0$

$\cancel{3x} - 2y = 6$

$-2y = 6$

$y = -3$

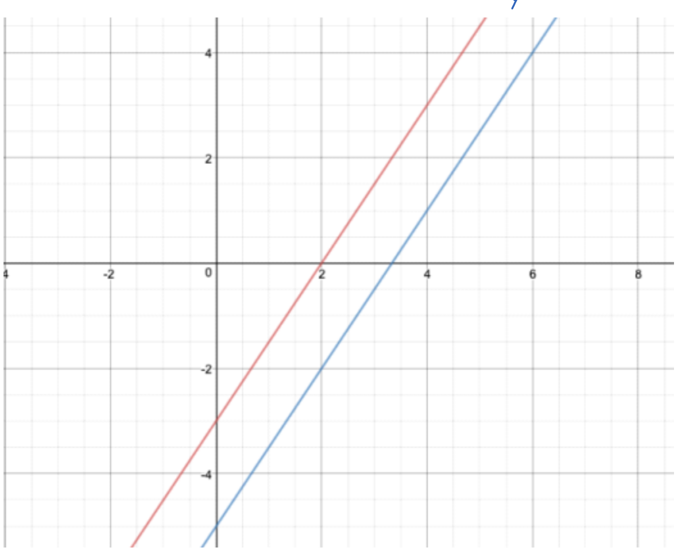


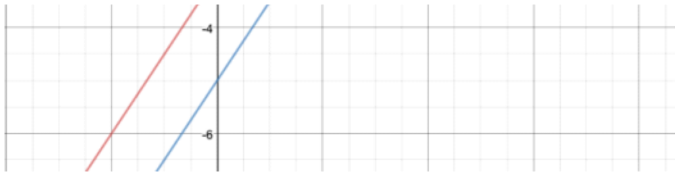
What about the following two lines?

$3x - 2y = 6$
$3x - 2y = 10$

What do you notice?

Sketching the above two lines we see:





So, there are three ways in which straight lines can meet:

- Meet only at one point (hence having one solution)
- Never meet (and hence parallel)
- Meet at an infinite number of points and hence the same line!

Theory done! Let's solve some.

Most people think that solving by substitution is the easiest and … for some equations it certainly is.
As with any area of Mathematics you need to ensure that you choose the best tool for the job.
To dig a tunnel out of school … would you use a spoon or a shovel?

Example: Solve the following simultaneous equations by substitution

$$y = 3x + 2$$
$$y = 5x - 6$$

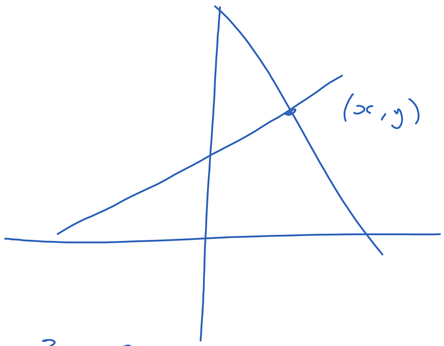
$y = 5x - 6$

$\cancel{3x} + 2 = 5x - 6$

$2 = 2x - \cancel{6}$

$8 = 2x$

$x = \underline{4}$



$y = 3x + 2$

$y = 3 \times 4 + 2$

$y = \underline{14}$

Check.

$y = 5x - 6$

$y = 20 - 6$

$y = \underline{14}$

Example: Solve the following simultaneous equations by substitution

$$3x + y = 3$$
$$2x - 2y = 10$$

$3x + y = 3$

$y = \underline{3 - 3x}$

$2x - 2y = 10$

$2x - 2(3 - 3x) = 10$

$2x - 6 + 6x = 10$

$8x - \cancel{6} = 10$

$8x = 16$

$x = \underline{2}$

$y = 3 - 3x$

$y = 3 - 3(2)$

$y = \underline{-3}$

$4 - (-6) = \underline{10}$ ✓

$\underline{(2, -3)}$

Example: Solve the following simultaneous equations by elimination

$$
\begin{array}{r}
4x + y = -1 \\
+ \quad 2x - y = -5 \\
\hline
6x = -6
\end{array}
$$

$x = \underline{-1}$

$4x + y = -1$

$4(-1) + y = -1$

$-4 + y = -1$

$y = 3$

$2x - y = -5$

$-2 - 3 = \underline{-5}$ ✓

$x = -1$

$-4 + y = -1$

$y = 3$

$(-1, 3)$

Example: Solve the following simultaneous equations by elimination

$$3x + 2y = 16 \quad \times 2$$
$$2x + 3y = 19 \quad \times 3$$

$$\cancel{+6x} + 4y = 32$$
$$- \quad \cancel{+6x} + 9y = 57$$
$$-5y = -25$$
$$y = 5$$

$3x + 2y = 16$

$3x + \cancel{10} = 16$

$3x = 6$

$x = 2$

$2x + 3y = 19$

$4 + 15 = 19$ ✓

$(2, 5)$

Using your CAS

It's important to note that, over there in Australia, this forms part of a CAS enabled course.
This means you need to be able to do things using both pencil and paper methods and using technology.
One of the most important functions (for now) is the SOLVE button.

Example: Solve the following simultaneous equations using your CAS

$$3x + 2y = 16$$
$$2x + 3y = 19$$

NOTE:
Always make sure you express you answers as a co-ordinate. You are solving two linear equations and, as such, you are finding the crossing point of two lines. They will always cross at a coordinate.