

Parallel and perpendicular lines

Thursday, 17 January 2019 5:06 PM

★ By the end of the lesson I would hope that you have an understanding of the concepts below which you can apply to a number of complex questions:

- Know what makes lines parallel and perpendicular
- How to identify from equations if lines are parallel and perpendicular
- Know how to use the fact that lines are parallel and perpendicular to find equations

RECAP

This whole chapter has been about the understanding and graphing of straight lines.

We move on to the idea of parallel and perpendicular lines.

This is a new concept and one which plays an important role in later sections of the course.

Parallel Lines

Parallel, by definition, means that "they shall never meet".

Hence, parallel lines are lines which will extend to infinity and beyond and never meet!

This suggests that they must have the same gradient.

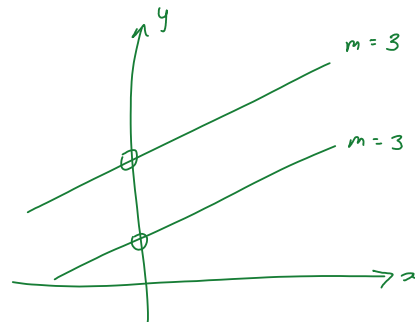
We can see if lines have the same gradient by comparing their equations.

$$\begin{aligned} y &= 3x - 2 \\ y &= 3x + 4 \end{aligned}$$

As this is in **gradient-intercept form** we can see, very clearly if the lines are parallel.

$$\begin{aligned} 3x + 2y &= 6 \\ 3x + 2y &= 8 \end{aligned}$$

As the above are in intercept form we can see clearly that the lines are parallel as the left hand side of the equation is the same.



Perpendicular lines

Perpendicular lines are those which cross each other at right angles.

This is very specific!

But, how do you tell if two lines have crossed at right angles?

Short cut:

Using the rule $m_1 m_2 = -1$

Now ... to be honest, I don't really use that form of the equation.

I tend to use the following:

If m_1 is the gradient of line 1 and m_2 is the gradient of the perpendicular I know that:

$$m_2 = -\frac{1}{m_1}$$

What does this mean?

Take the original gradient, reciprocate it (flip it upside down) and change the sign.

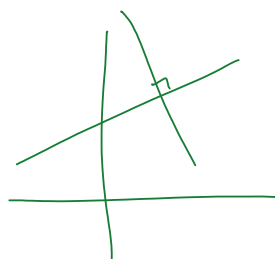
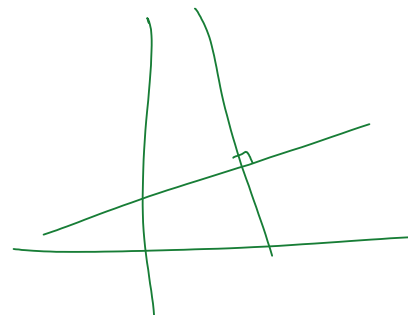
Example:

Find the gradient of a line which is perpendicular to a line with gradient 2.

$$m_1 = 2$$

$$m_2 = -\frac{1}{2}$$

$$\begin{aligned} 3x + 2y &= 6 \\ 2y &= -3x + 6 \\ y &= \left(-\frac{3}{2}\right)x + 3 \\ 3x + 2y &= 8 \\ 2y &= -3x + 8 \\ y &= \left(-\frac{3}{2}\right)x + 4 \end{aligned}$$



Example:

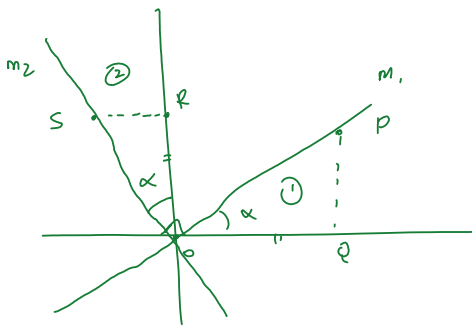
Find the gradient of a line which is perpendicular to a line with gradient $-\frac{1}{3}$.

$$m_1 = -\frac{1}{3}$$

$$m_2 = \frac{3}{1} = 3$$

Theory behind the formula works!

Not that you will ever probably need this, but it makes for a nice fill before we do some examples!



$$m_1 = \frac{PQ}{OQ}$$

$$m_2 = \frac{OR}{-RS}$$

↑

$$\frac{1}{m_2} = -\frac{RS}{OR}$$

$$-\frac{1}{m_2} = \frac{RS}{OR}$$

$$\tan \alpha = \frac{PQ}{OQ} \quad m_1 = \tan \alpha$$

$$\tan \alpha = \left(\frac{RS}{OR} \right) = -\frac{1}{m_2}$$

$$-\frac{1}{m_2} = m_1$$

$$m_1 m_2 = -1$$

Examples

Let's put this into practice!

Example 1:

Taken from the Cambridge Essentials Textbook Series:

Find the equation of the straight line which passes through (1, 2) and is parallel to the line with equation $2x - y = 4$

$$y - y_1 = m(x - x_1)$$

$$y - 2 = m(x - 1)$$

$$y - 2 = 2(x - 1)$$

$$y - 2 = 2x - 2$$

$$y = 2x$$

$$2x - y = 4$$

$$-y = -2x + 4$$

$$y = 2x - 4$$

Example 2:

Taken from the Cambridge Essentials Textbook Series:

Find the equation of the straight line which passes through (1, 2) and is perpendicular to the line with equation $2x - y = 4$

$$y - y_1 = m(x - x_1)$$

$$y - 2 = m(x - 1)$$

$$y - 2 = -\frac{1}{2}(x - 1)$$

$$2y - 4 = -(x - 1)$$

$$2y - 4 = -x + 1$$

$$2y = -x + 5$$

$$y = -\frac{x}{2} + \frac{5}{2}$$

$$2x - y = 4$$

$$y = 2x - 4$$

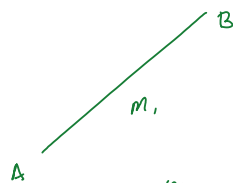
$$m_1 = 2 \quad m_2 = -\frac{1}{2}$$

$$y = -\frac{1}{2}x + \frac{5}{2}$$

Example 3:

Taken from the Cambridge Essentials Textbook Series:

The coordinates of the vertices of a triangle ABC are $A(0, -1)$, $B(2, 3)$ and $C(3, -2\frac{1}{2})$. Show that the side AB is perpendicular to the side AC .



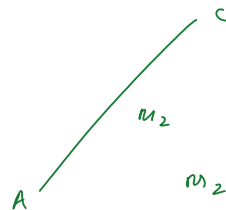
$$m_1 = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{3 - (-1)}{2 - 0}$$

$$= \frac{4}{2}$$

$$m_1 = 2$$

$$m_1 m_2 = -1$$



$$m_2 = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-2\frac{1}{2} - (-1)}{3 - 0}$$

$$= -\frac{1\frac{1}{2}}{3}$$

$$= -\frac{3/2}{3}$$

$$m_2 = -\frac{1}{2}$$

$$-\frac{3}{2} \div 3$$

$$= -\frac{3}{2} \times \frac{1}{3}$$