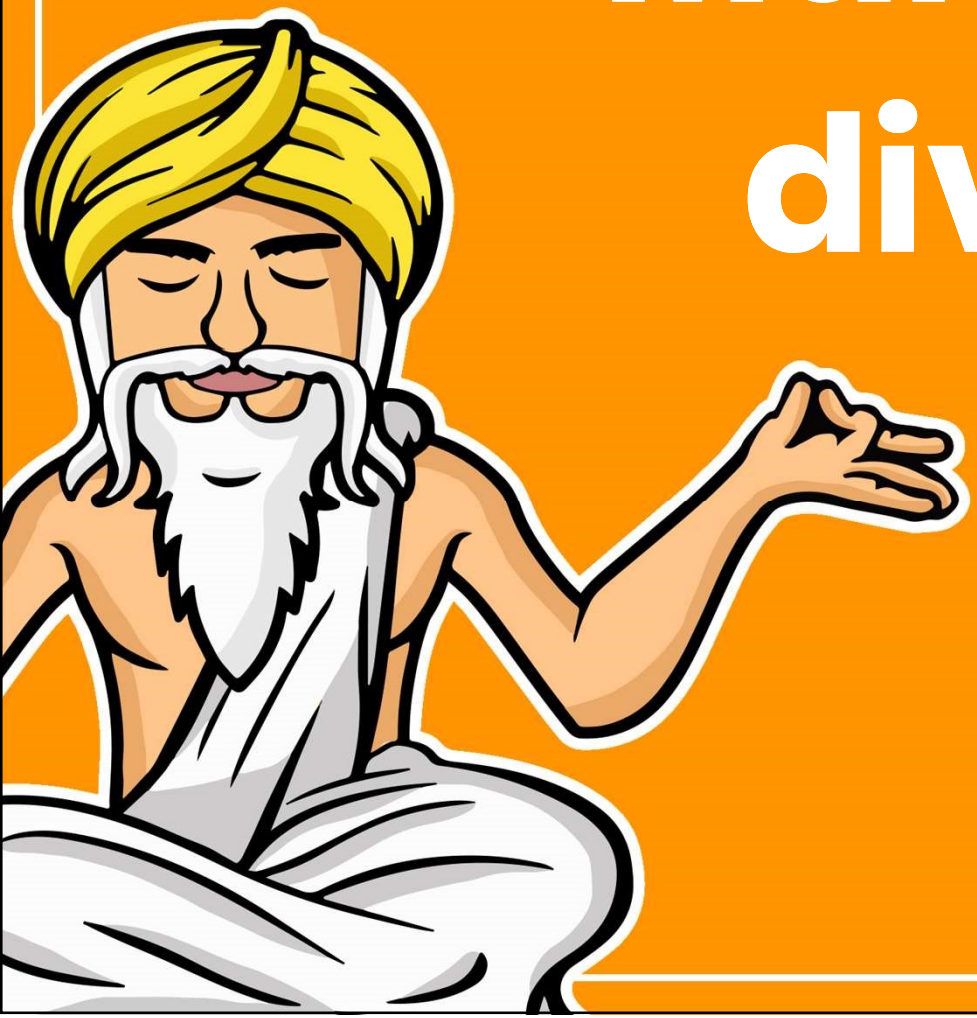
Multiplying and dividing surds



Year 10 Maths
Advanced

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Learning Objectives

By the end of the lesson I would hope that you have an understanding and be able to apply to questions the following concepts:

- Understand how to multiply and divide surds
- Apply the distributive law to brackets involving surds

$2\sqrt{2} + \sqrt{8}$

$1\sqrt{3} + 3\sqrt{3}$

$= 5\sqrt{3}$

$\frac{1}{2} + \frac{1}{3}$

$\frac{1}{2} \times \frac{1}{3}$

Recap

In previous lessons we have looked at how we can add and subtract surds by ensuring that they have the same positive integer under the root sign. This generally requires us to simplify the surds.

We have also looked at the idea shown below:

$$\sqrt{xy} = \sqrt{x} \times \sqrt{y}$$

This can also be reversed to show:

$$\sqrt{x} \times \sqrt{y} = \sqrt{xy}$$

When we multiply or divide two surds we do not need to ensure the positive value under the root is the same. We do need to ensure we simplify any resultant multiplication or division!

$\sqrt{xy} = \sqrt{x} \times \sqrt{y}$

$\sqrt{x} \times \sqrt{y} = \sqrt{xy}$

$\sqrt{\frac{x}{y}} = \frac{\sqrt{x}}{\sqrt{y}} =$

$\frac{\sqrt{x}}{\sqrt{y}} = \sqrt{\frac{x}{y}}$

Let's jump right into some examples

Simplify the following:

- $\sqrt{2} \times \sqrt{3}$
- $2\sqrt{3} \times 3\sqrt{15}$
- $(2\sqrt{5})^2$

$\sqrt{2} \times \sqrt{3} = \sqrt{2 \times 3} = \sqrt{6}$

$2\sqrt{3} \times 3\sqrt{15} = 2 \times 3 \times \sqrt{3} \times \sqrt{15}$

$= 6\sqrt{3 \times 15}$

$= 6\sqrt{45}$

$= 6\sqrt{9 \times 5}$

$= 6 \times 3 \times \sqrt{5}$

$= 18\sqrt{5}$

$(ab)^2 = a^2b^2$

$(2\sqrt{5})^2$

$2^2 \times (\sqrt{5})^2$

$= 4 \times 5$

$= 20$

Examples have been extracted, with permission, from the Cambridge Essential Mathematics Series of textbooks

Division is a tricky one!

When we divide surds we need to be aware that there are two rules in operation; numbers outside of the surds and numbers under the surds.

Remember the basic rules of fractions apply.

When you can cancel down you must.

$$\frac{\cancel{2}\sqrt{5}}{\cancel{10}_{5}} = \frac{\sqrt{5}}{5}$$

Division is a tricky one!

When we divide surds we need to be aware that there are two rules in operation; numbers outside of the surds and numbers under the surds.

We can rewrite division sums using the following:

$$\sqrt{x} \div \sqrt{y} = \sqrt{\frac{x}{y}}$$

Once written in the above format you can then cancel down as you would normally.

Don't forget that the questions will try and trick you!

$a^6 \div a^2 = a^4$

$\sqrt{x} \div \sqrt{y} = \frac{\sqrt{x}}{\sqrt{y}} = \sqrt{\frac{x}{y}}$

Examples of dividing surds

Simplify these surds

- $-\sqrt{10} \div \sqrt{2}$
- $\frac{12\sqrt{18}}{3\sqrt{3}}$

$$\bullet \quad \frac{-\sqrt{10}}{\sqrt{2}} = -\sqrt{\frac{10}{2}}$$

$$= -\sqrt{5}$$

$$\bullet \quad \frac{\overset{4}{\cancel{12}}\sqrt{18}}{\underset{1}{\cancel{3}}\sqrt{3}} = \frac{4\sqrt{9\times 2}}{\sqrt{3}} = \frac{4\times 3\times\sqrt{2}}{\sqrt{3}}$$

$$= \frac{12\sqrt{2}}{\sqrt{3}}$$

The distributive law

Multiplying out brackets is awesome and is going to form a significant part of the Year 10 course (if it hasn't already!)

The distributive law states:

$$a(b+c) = ab + ac$$

$a(b+c)$

$= ab + ac$

$\sqrt{3}(\sqrt{2}+4)$

$= \sqrt{6} + 4\sqrt{3}$

The distributive law with surds

We already know how to multiply surds from a previous set of slides. We know how to simplify them.

The important thing to remember with multiplying surds is that you need to multiply the numbers in front of the root **and** the numbers under the root.

For example:

$3\sqrt{3} \times 4\sqrt{2}$ $= 3 \times 4 \times \sqrt{3} \times \sqrt{2}$

$= 12\sqrt{6}$

Examples with the distributive law and surds

Use the distributive law to expand the following and then simplify the surds where necessary.

- $\sqrt{3}(3\sqrt{5} - \sqrt{6})$
- $3\sqrt{6}(2\sqrt{10} - 4\sqrt{6})$

$(\sqrt{6})^2$

$\sqrt{6} \times \sqrt{6}$

$= \sqrt{36}$

$= 6$

$\sqrt{3}(3\sqrt{5} - \sqrt{6})$

$= 3\sqrt{15} - \sqrt{18}$

$= 3\sqrt{15} - \sqrt{9 \times 2}$

$= 3\sqrt{15} - 3\sqrt{2}$

$3\sqrt{6}(2\sqrt{10} - 4\sqrt{6})$

$= 6\sqrt{60} - 12 \times 6$

$= 6\sqrt{4 \times 15} - 72$

$= 6 \times 2 \times \sqrt{15} - 72$

$= 12\sqrt{15} - 72$