Irrational numbers including surds

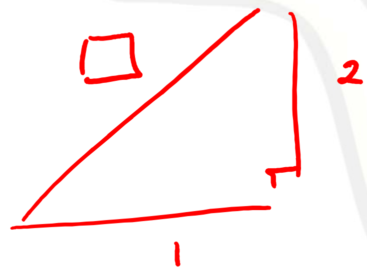
Year 10 Maths
Advanced

Learning Objectives

By the end of the lesson I would hope that you have an understanding and be able to apply to questions the following concepts:

- Understand what an irrational number is
- Understand what a rational number is
- Understand what a surd is
- Be able to identify numbers as rational or irrational
- Be able to simplify surds

$3.1479623\ldots$



Recap

This is the first in a new series of videos relating to Indices and Surds at a Year 10 level. Although this work may have been covered previously I will look afresh at the ideas.

Why would we want to express numbers using indices and surds?

Well, it actually makes life much easier for us.

Rather than writing decimal numbers rounded to a certain number of decimal places (and hence introducing rounding errors) we can keep a version of the number.

WE can express numbers like 1, 000, 000, 000, 000 in a different form to sav us from writing all those zeros!

Aren’t you being irrational?

Before we get to what a surd is, let’s talk about rational and irrational numbers first.

A **rational number** which can be expressed as a fraction. Note: Whole numbers are rational!

An **irrational number** is one which cannot be expressed as a fraction.

$\frac{1}{2}$ $\frac{2}{3}$ $\frac{4}{7}$ $\frac{3}{1}$

137% $1 \cdot 37$ $1^{37}/_{100}$

π $\sqrt{2}$ $\sqrt{3}$ $\sqrt{5}$

Examples of rational and irrational numbers

Express each number as a decimal and decide if it is rational or irrational:

- $-\sqrt{3}$
- 137%
- $\frac{3}{7}$

$-\sqrt{3} = -1.732050808\ldots$

$137\% = 1.37$

$1\frac{37}{100}$

$^{137}/_{100}$

$\frac{3}{7} = 0.428571428\ldots$

$= 0.\overline{428571}$

Examples have been extracted, with permission, from the Cambridge Essential Mathematics Series of textbooks

Irrational numbers with a root sign

An irrational number is one which cannot be expressed as a fraction.

A surd is an irrational number with a **root sign**.

There are many roots! Not just square roots.

We must understand how to read a root sign and what it means.

$\sqrt{2}$ $\quad \cancel{\sqrt{4}} = \frac{2}{1}$

$\sqrt{3}$

$\sqrt{5}$ $\quad \sqrt[3]{9}$

The square of a square root

When we square a number, we multiply it by itself.

When we take the square root of a number, we are finding which two numbers multiply to give the value under the square root.

Taking the square root and squaring something are opposite operations. Hence, if we square a square root, we end up back where we started.

$$2^2 = 2 \times 2 = 4$$

$$(\sqrt{2})^2 = \sqrt{2} \times \sqrt{2} = 2$$

$$\sqrt{2^2} = 2 \qquad (\sqrt{3})^2 = 3$$

Index laws for surds

In Year 9 you will have covered the basic rules behind powers (indices) and how we can manipulate them.

There are also rules for roots which can be shown to be true.

$$\sqrt{xy} = \sqrt{x} \times \sqrt{y}$$

$$\sqrt{\frac{x}{y}} = \frac{\sqrt{x}}{\sqrt{y}}$$

$x^2 \times x^3 = x^5$

$\sqrt{xy} = \sqrt{x} \times \sqrt{y}$

$\sqrt{2 \times 3} = \sqrt{2} \times \sqrt{3}$

$\sqrt{\frac{7}{4}} = \frac{\sqrt{7}}{\sqrt{4}}$

$= \frac{\sqrt{7}}{2}$

$\sqrt{\frac{x}{y}} = \frac{\sqrt{x}}{\sqrt{y}}$

Simplifying surds

Like fractions, surds can be simplified using the rules shown below.

$$\boxed{\sqrt{xy} = \sqrt{x} \times \sqrt{y}}$$

$$\sqrt{\frac{x}{y}} = \frac{\sqrt{x}}{\sqrt{y}}$$

If a number under a root sign can be split into the product of a square number and another number, we can write the surd in a simplified form

Note: Important square numbers are 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225.

400

$$\sqrt{20} = \sqrt{4 \times 5}$$
$$= \sqrt{4} \times \sqrt{5}$$
$$= \underline{\underline{2\sqrt{5}}}$$

$$\sqrt{72} = \sqrt{36 \times 2}$$
$$= \sqrt{36} \times \sqrt{2}$$
$$= \underline{\underline{6\sqrt{2}}}$$

Examples of simplifying surds

Simplify the following:

- $\sqrt{32}$
- $3\sqrt{200}$
- $\frac{5\sqrt{40}}{6}$
- $\sqrt{\frac{75}{9}}$

$$\sqrt{\frac{75}{9}} = \frac{\sqrt{75}}{\sqrt{9}} = \frac{\sqrt{25\times3}}{\sqrt{9}} = \frac{\sqrt{25}\times\sqrt{3}}{\sqrt{9}} = \frac{5\sqrt{3}}{3}$$

$$\sqrt{32} = \sqrt{16\times2} = \sqrt{16}\times\sqrt{2} = 4\sqrt{2}$$

$$3\sqrt{200} = 3\times\sqrt{100\times2} = 3\times\sqrt{100}\times\sqrt{2} = 3\times10\times\sqrt{2} = 30\sqrt{2}$$

$$\frac{5\sqrt{40}}{6} = \frac{5\times\sqrt{4\times10}}{6} = \frac{5\times\sqrt{4}\times\sqrt{10}}{6} = \frac{5\times\cancel{2}\times\sqrt{10}}{\cancel{6}\,3} = \frac{5\sqrt{10}}{3}$$

Examples have been extracted, with permission, from the Cambridge Essential Mathematics Series of textbooks

Doing things backwards

Remember, what we do forwards in Mathematics we also have to be able to do backwards.

Hence, if we have the following simplified surd:

$$3\sqrt{5}$$

We can turn it into an "un-simplified" version or "as a square root of a positive integer".

Note: Integer means a whole number

$3\sqrt{5}$

$= \sqrt{9} \times \sqrt{5}$

$= \sqrt{9 \times 5}$

$= \underline{\sqrt{45}}$

Examples of doing things backwards

Express these surds as a square root of a positive integer.

- $2\sqrt{5}$
- $7\sqrt{2}$

$2\sqrt{5}$

$= \sqrt{4} \times \sqrt{5}$

$= \sqrt{4 \times 5}$

$= \sqrt{20}$

$7\sqrt{2}$

$= \sqrt{49} \times \sqrt{2}$

$= \sqrt{98}$

Examples have been extracted, with permission, from the Cambridge Essential Mathematics Series of textbooks

Thanks for watching

All videos are available to view at **www.maffsguru.com**
Lesson notes can be downloaded too

Please visit **www.youtube.com/maffsguru** and subscribe