



Graphs of polynomials

Year 10 Mathematics

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Learning Objectives

By the end of the lesson, I would hope that you have an understanding and be able to apply to questions the following concepts:

- To know the shape of the graphs of the basic polynomials $y = x^3$ and $y = x^4$
- To know the shape of a graph of a cubic polynomial with three different factors
- To be able to find the axis intercepts of a cubic graph
- To know how to use the shape and intercepts to sketch a cubic graph



Recap of past learning

Everything we do in Mathematics is for a reason even though it might not seem like it!

Polynomials are graphs. They are algebraic representations of the graph. For every polynomial we are given, we can use the CAS or Desmos to show what it actually looks like. This can be really important in later sections of the course.

So, we're going to look at how we can graph polynomials.



Plotting graphs is really important

In later years you are going to be expected to sketch graphs. You will need to do this in a particular way. You will be asked, in questions, to find:

- X-intercepts
- Y-intercepts
- Turning points
- Points of inflection

And other things. Using these points and a good grasp of the maths we are doing you will be able to sketch a pretty close approximation to the graph you are being asked to consider.

For now, we are simply going to create a table of values and then use this to plot the graph.

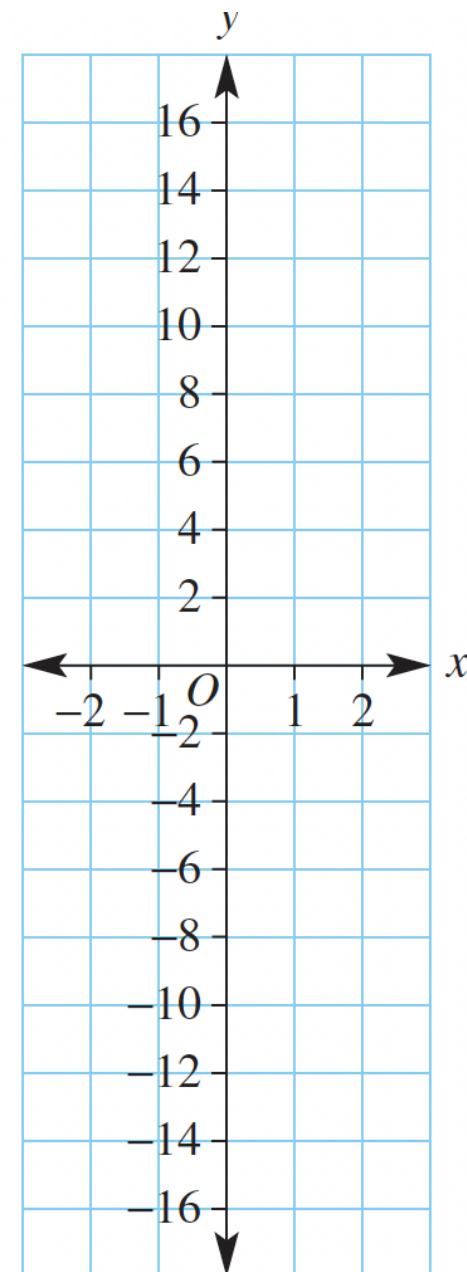


Plotting graphs is really important

Let's plot the graphs of $y = x^3$ and $y = x^4$

x	-2	-1	-1/2	0	1/2	1	2
$y = x^3$	-8	-1	$-\frac{1}{8}$	0	$\frac{1}{8}$	1	8
$y = x^4$	16	1					

$$\left(-\frac{1}{2}\right)^3$$



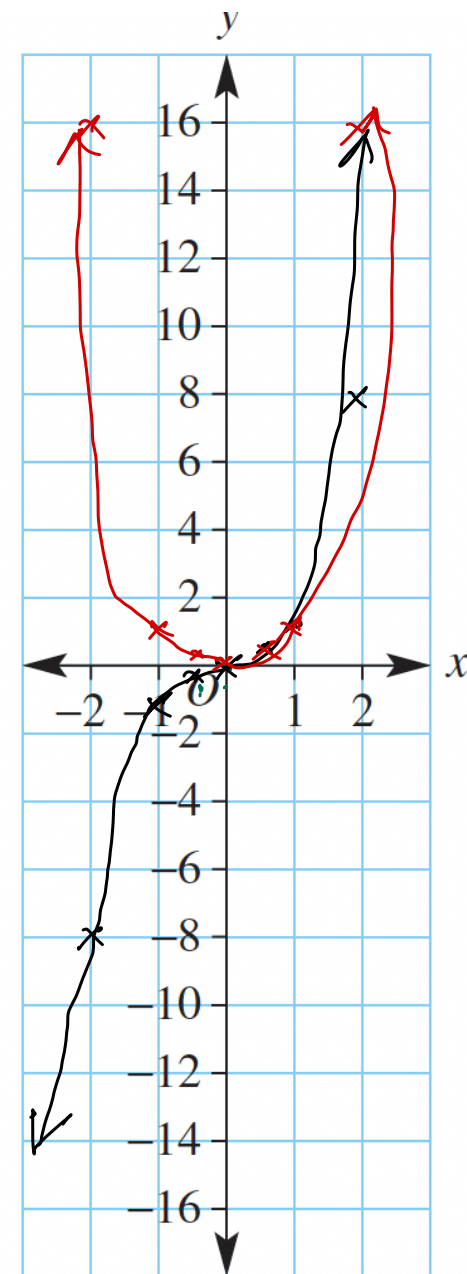
Note: Always make the graph as large as you can. It improves accuracy

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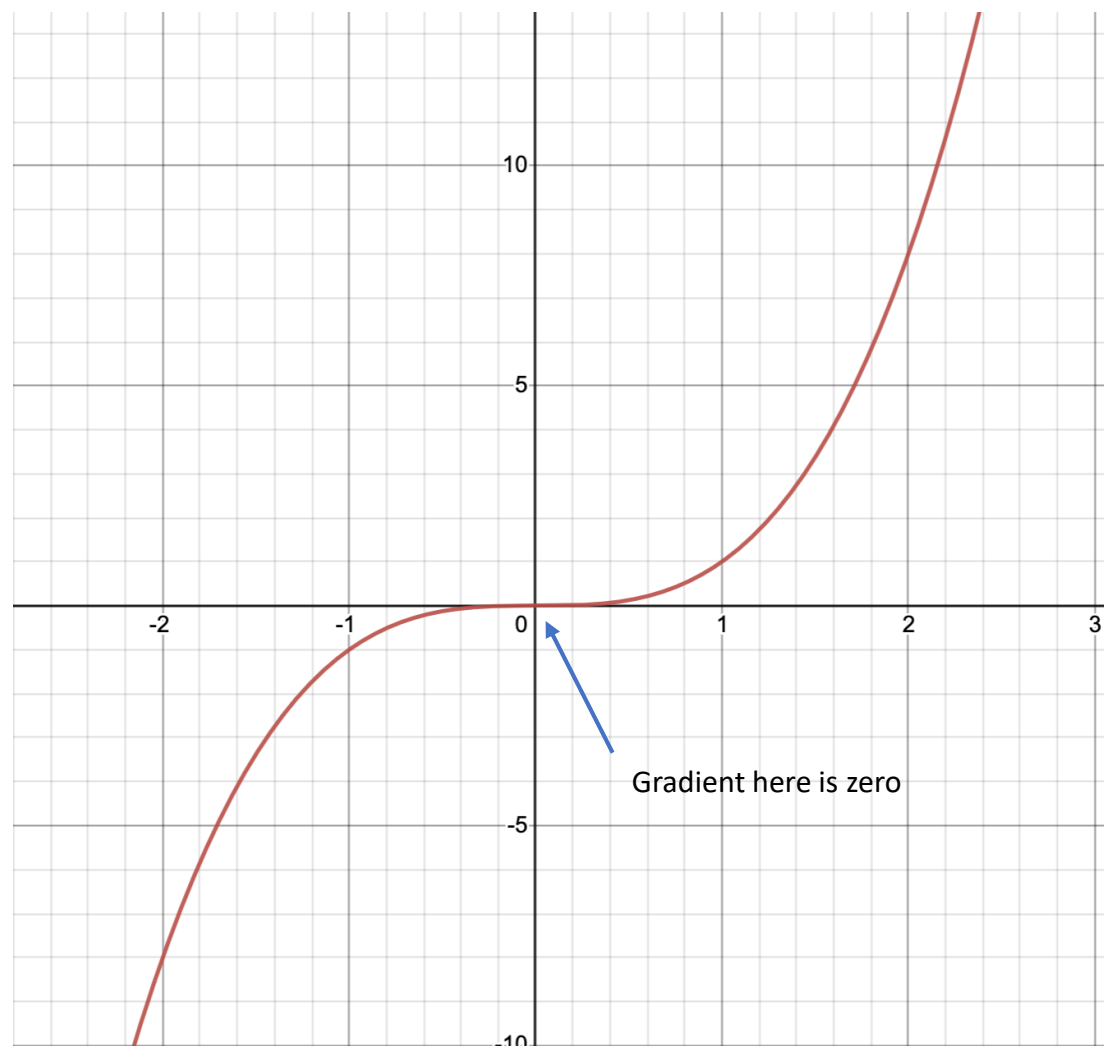
x	-2	-1	-1/2	0	1/2	1	2
$y = x^3$	-8	-1	$-\frac{1}{8}$	0	$\frac{1}{8}$	1	8
$y = x^4$	16	1	$\frac{1}{16}$	0	$\frac{1}{16}$	1	16

Note: Always make the graph as large as you can. It improves accuracy



Graphs of the more “common” polynomials: Cubic

Knowing the general shape and characteristics of polynomials is really helpful going forward. These should be in your summary book or notes. If you can, memorise them! It makes life so much easier.

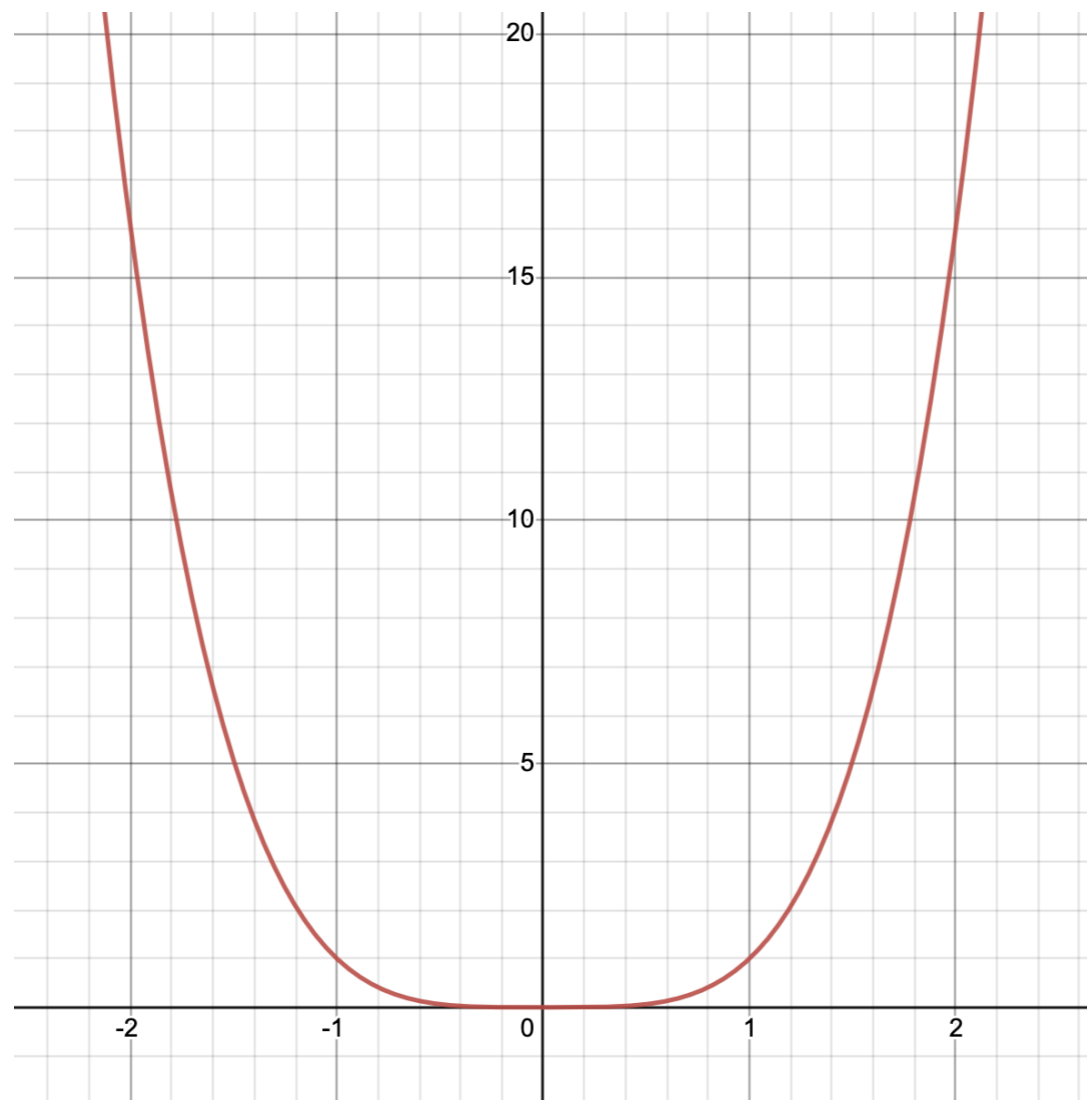


The cubic $y = x^3$



Graphs of the more “common” polynomials: Quartic

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The quartic $y = x^4$

Looks like a quadratic!

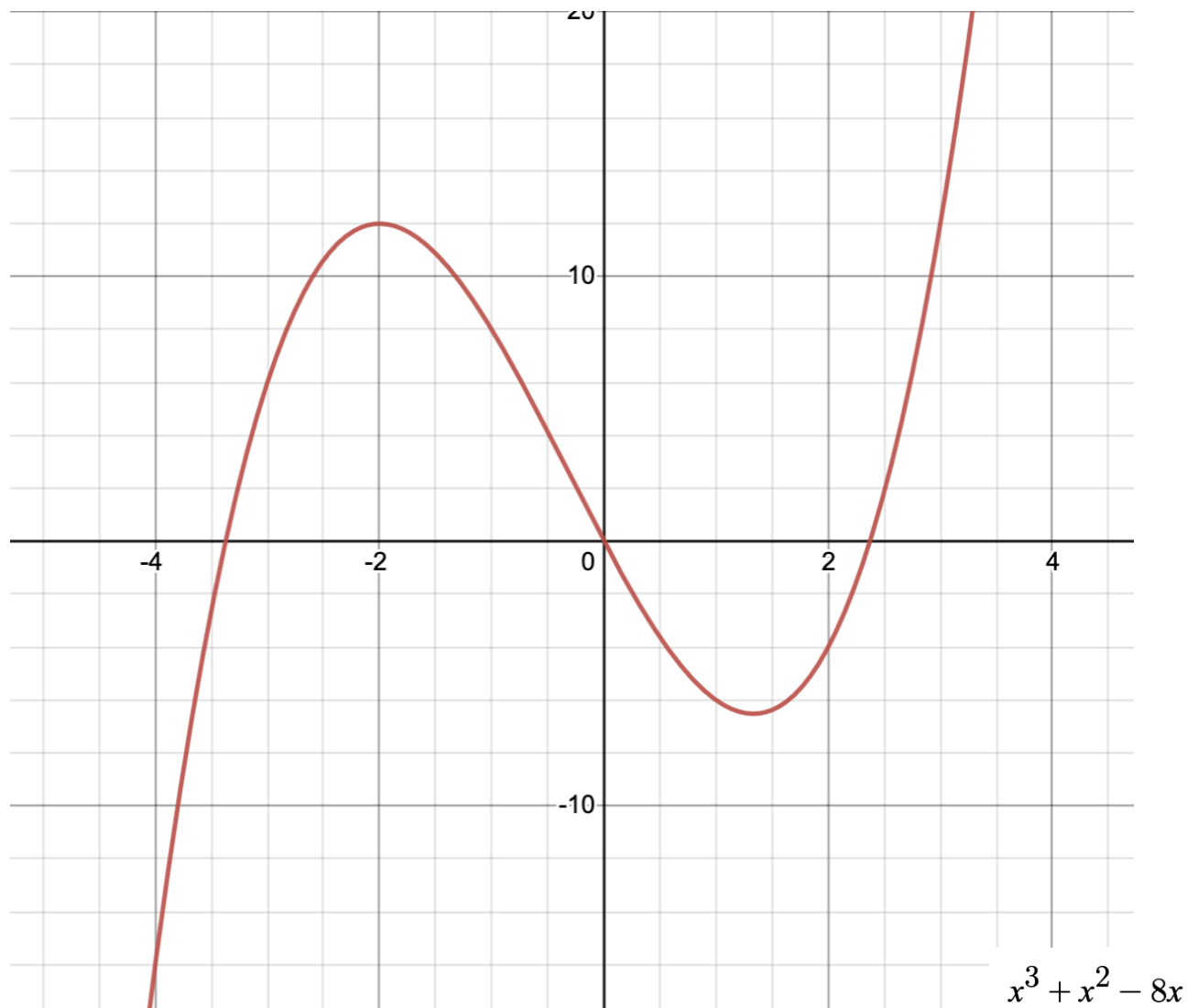


Graphs of the more “common” polynomials: Positive Cubic

Cubics can look different from that shown before. When we have more terms, it looks more like that shown on the right.

Things to note:

Turning points are NOT symmetrical.
There are generally three factors and hence three points where they touch or cross the x-axis.

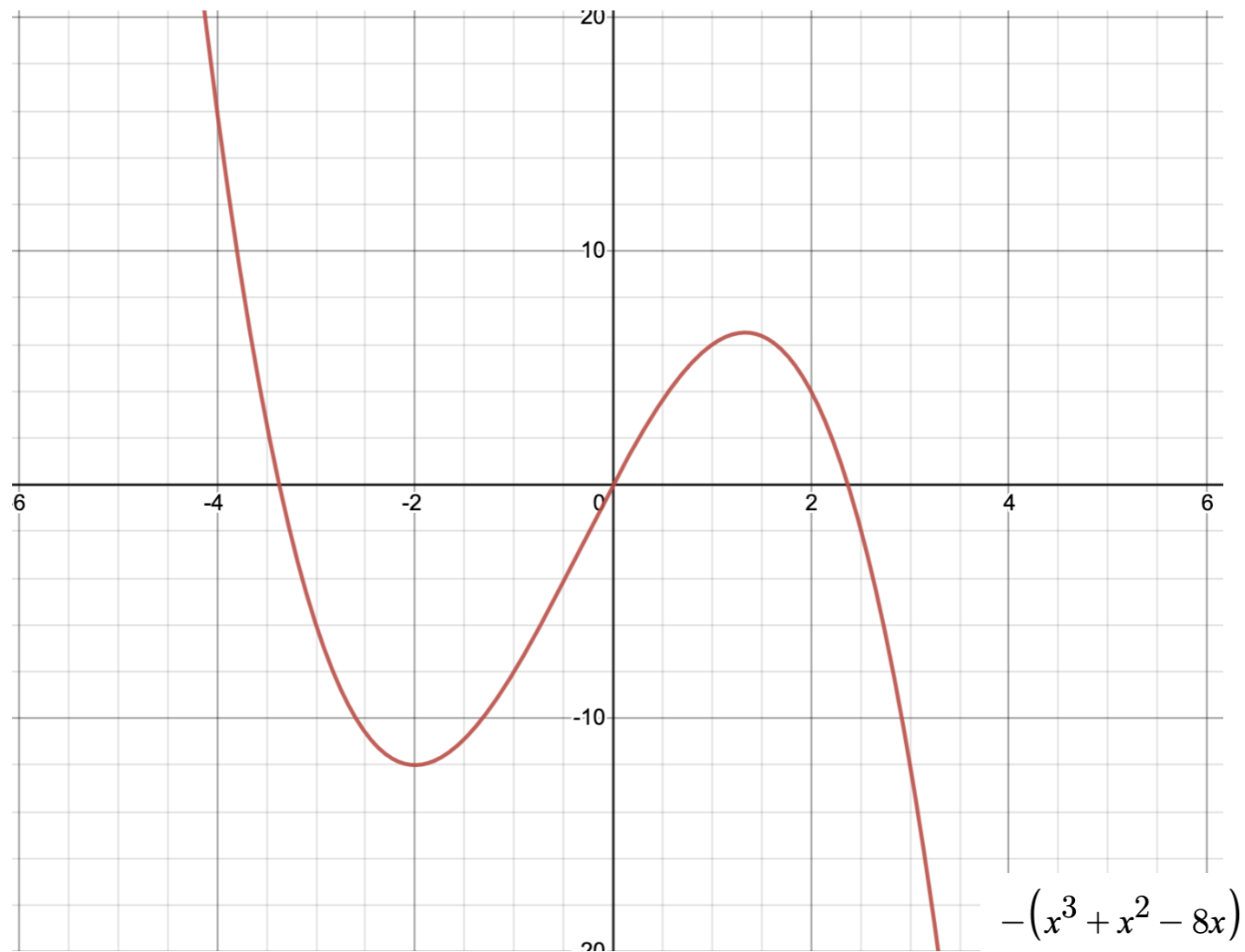


Graphs of the more “common” polynomials: Negative Cubic

Cubics can look different from that shown before. When we have more terms, it looks more like that shown on the right.

Things to note:

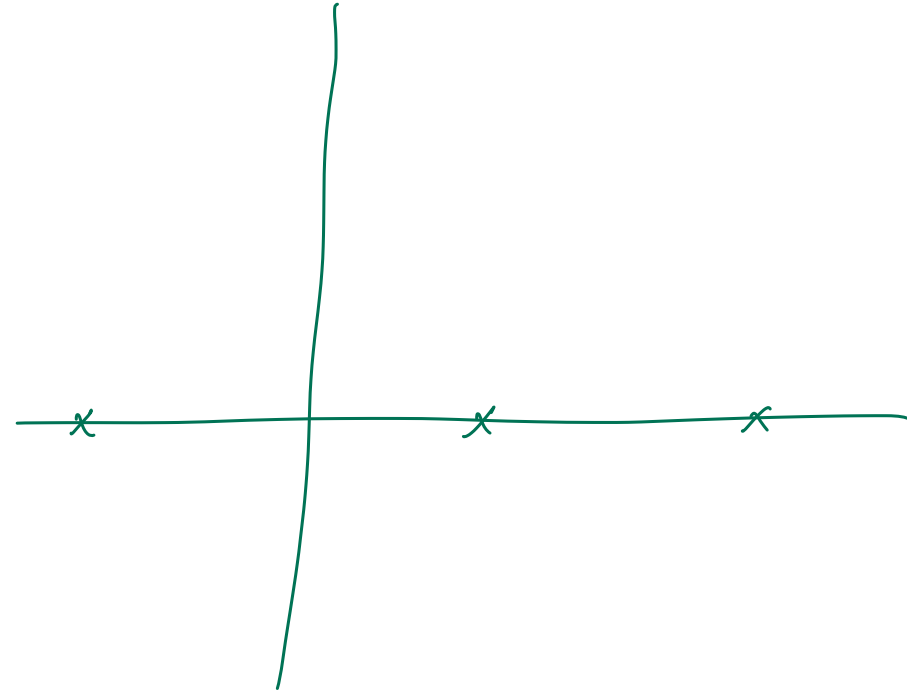
Turning points are NOT symmetrical. There are generally three factors and hence three points where they touch or cross the x-axis.



Sketching cubics: A recipe

Everything from now on will have some form of recipe. Here is the one you need to sketch a cubic.

1. Find the three x-intercepts
 1. Polynomial division
 2. Factorise
 3. Use Null Factor Law
2. Find the y-intercept (by putting $x = 0$)
3. Connect points to sketch either the positive or negative form



Algebra trickery

Mathematics is a big fat trick. We can try and trick you in the following ways:

$$(x - 3)(x + 4)$$



If I factorise out a negative 1 this means I am dividing by -1.
This makes the following also true

$$-(3 - x)(x + 4)$$

$$-1(-x + 3)(x + 4)$$

$$-1(3 - x)(x + 4)$$



Example: Sketching a cubic graph

Sketch the graphs of the following by finding the x - and y -intercepts.

a $y = (x + 2)(x - 1)(x - 3)$

b $y = -x(x + 3)(x - 2)$

Note that these are already in factor form making it easy to sketch

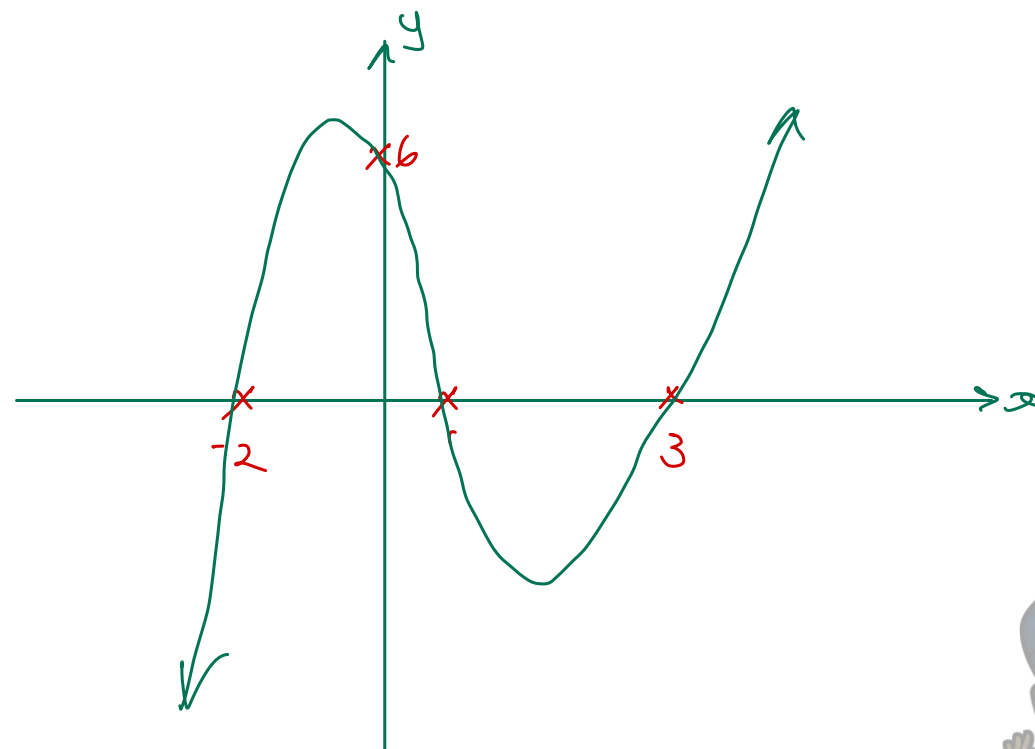
$$\begin{aligned} x &= 0 \\ y &= 2 \times 1 \times -3 \\ &= -6 \end{aligned}$$

$$(x + 2)(x - 1)(x - 3) = 0$$

$$\begin{aligned} x + 2 &= 0 \\ x &= \underline{\underline{-2}} \end{aligned}$$

$$\begin{aligned} x - 1 &= 0 \\ x &= \underline{\underline{1}} \end{aligned}$$

$$\begin{aligned} x - 3 &= 0 \\ x &= \underline{\underline{3}} \end{aligned}$$



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$$(x + 0)(x + 3)(x - 2) = 0$$

$$\begin{aligned}\therefore x + 0 &= 0 \\ x &= 0\end{aligned}$$

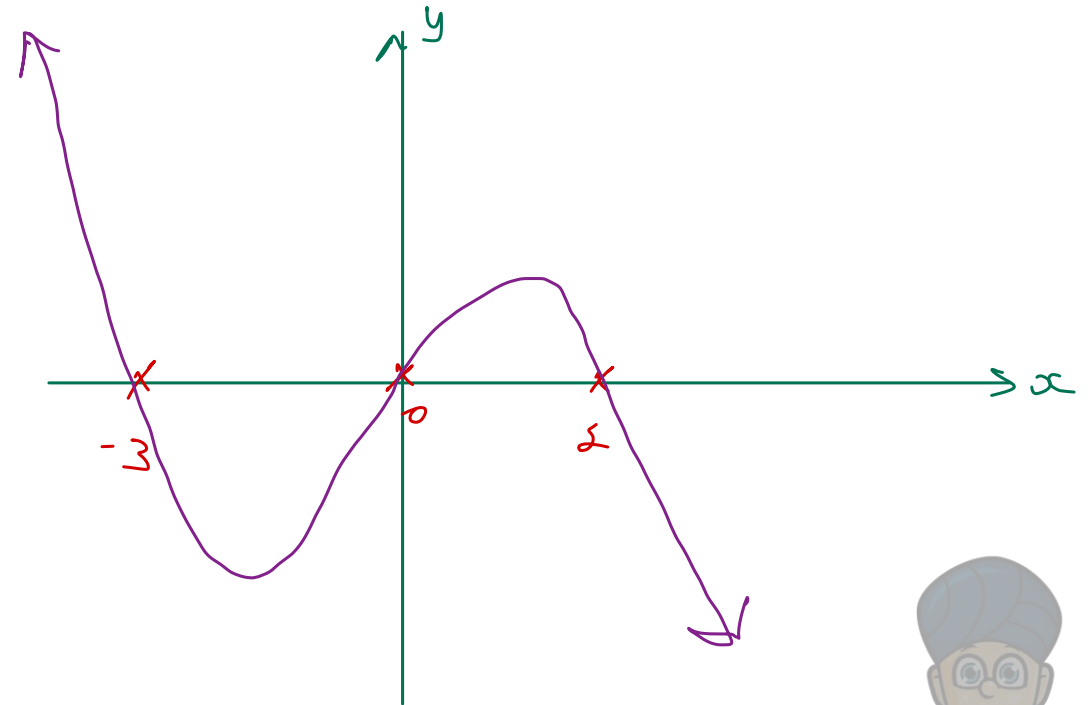
$$\begin{aligned}x + 3 &= 0 \\ x &= -3\end{aligned}$$

$$\begin{aligned}x - 2 &= 0 \\ x &= 2\end{aligned}$$

$$x = 0$$

$$y = 0$$

$$y = -0 \times 3 \times -2$$



Remember you can check with your CAS

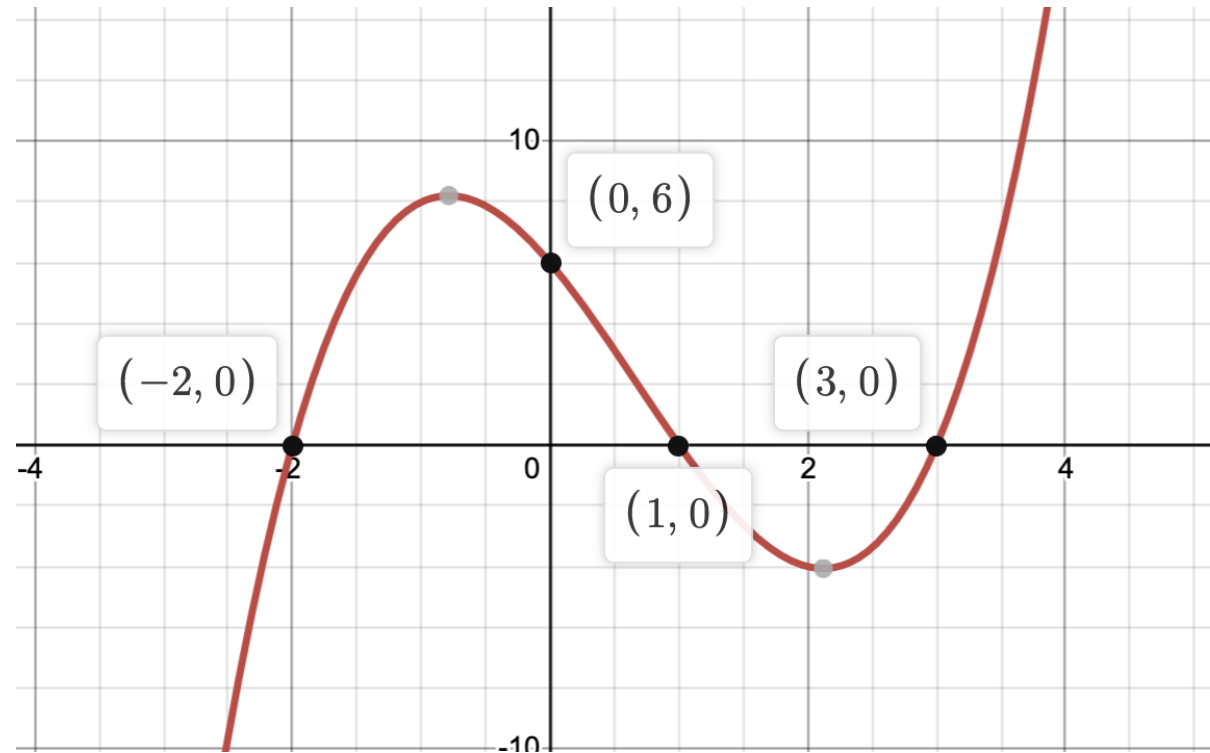
Sketch the graphs of the following by finding the x - and y -intercepts.

a $y = (x + 2)(x - 1)(x - 3)$

b $y = -x(x + 3)(x - 2)$

Checking that it is correct is very different from using the CAS to cheat.

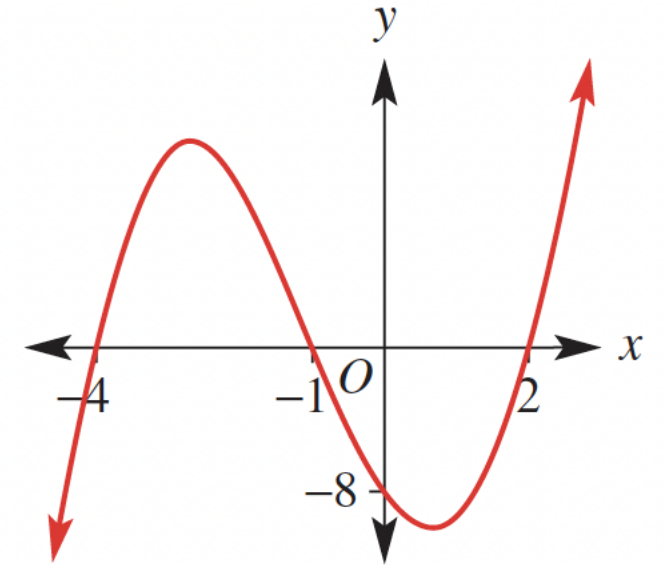
These questions in the exam will expect you to show working out.



Finding a cubic rule from a graph

When given the graph shown on the right, we can find a cubic rule by reversing the learning we have acquired. It is important to note that there are many, many graphs which will have the same crossing points but different expressions.

$$\begin{array}{lll} x = -4 & x = -1 & x = 2 \\ x + 4 = 0 & x + 1 = 0 & x - 2 = 0 \end{array}$$
$$(x + 4)(x + 1)(x - 2)$$



Tricks, tricks and more tricks

Your understanding of the algebra is going to be really important. Remember that we can write things in different ways to try and trick you.

For example:

$$y = x(x - 3)^2$$

$$x(x - 3)(x - 3)$$

Are these cubics?

$$y = (2 - x)(x + 1)^2$$

$$-1(x - 2)(x + 1)^2$$

Always try and write them in a form which looks familiar and then you won't make mistakes



Using long division to sketch a cubic

Let's use the whole method now to sketch $y = x^3 + 4x^2 + x - 6$

- Find the three x-intercepts
 - Polynomial division
 - Factorise
 - Use Null Factor Law
- Find the y-intercept (by putting $x = 0$)
- Connect points to sketch either the positive or negative form

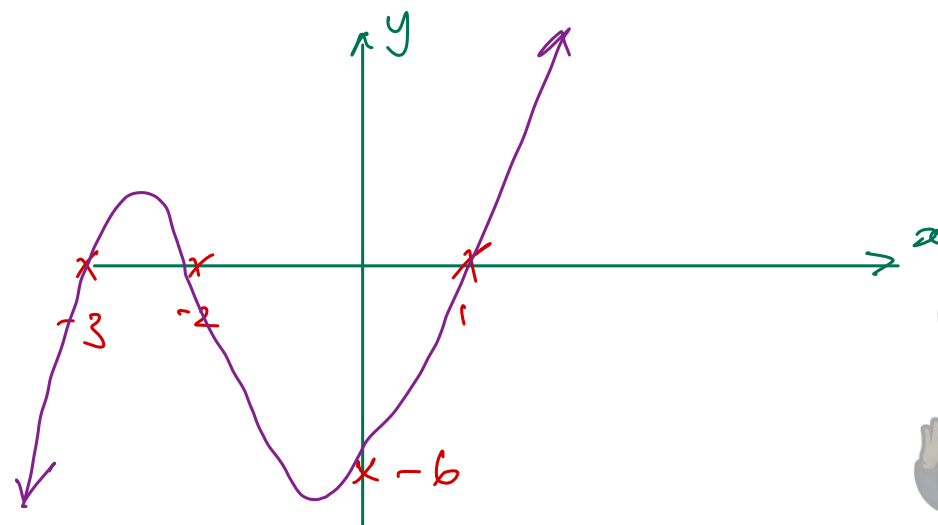
$$\begin{array}{r}
 x = 1 \\
 x - 1 = 0 \\
 x - 1 \overline{) \begin{array}{r} x^3 + 4x^2 + x - 6 \\ - (x^3 - x^2) \\ \hline 5x^2 + x \\ - (5x^2 - 5x) \\ \hline 6x - 6 \\ - (6x - 6) \\ \hline 0 \end{array}} \\
 \hline
 \end{array}$$

$$y = (x - 1)(x^2 + 5x + 6)$$

$$\begin{array}{r}
 6 \\
 1 \overline{) 6} \\
 2 \overline{) 3}
 \end{array}$$

$$(x + 2)(x + 3)$$

$$y = (x - 1)(x + 2)(x + 3)$$



$$x - 1 = 0$$

$$x + 2 = 0$$

$$x + 3 = 0$$

$$x = 1$$

$$x = -2$$

$$\underline{\underline{x = -3}}$$



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