



The remainder and factor theorems

Year 10 Mathematics

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Learning Objectives

By the end of the lesson, I would hope that you have an understanding and be able to apply to questions the following concepts:

- To know how to use the remainder theorem to find the remainder when a polynomial is divided by a linear expression
- To understand that a remainder of zero implies that a divisor is a factor
- To be able to use the factor theorem to decide if a linear expression is a factor of a polynomial



Recap of past learning

If you have just come from the last lesson looking at how to divide polynomials you might feel like the world is about to end ... well ... it's not! But it was important to come up with a process to do it and an understanding of what it means to be a quotient and a remainder.

We know, from basic Year 5, 6 and 7 maths that when a number divides into another a **whole number of times** there will be no remainder. More importantly the remainder will be zero. This means the divisor is a **factor** of the dividend.

When it doesn't divide in a whole number of times there will be a remainder.

This becomes really important now!

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Dividend

$$\frac{x^3 - x^2 + x - 1}{x + 2} = x^2 - 3x + 7 - \frac{15}{x + 2}$$

Divisor

Quotient

Remainder



Long division can help us know if something is a factor or not

As the title says, we can find out if something is a factor or something else by testing to see if there is a remainder or not. Is there another way?

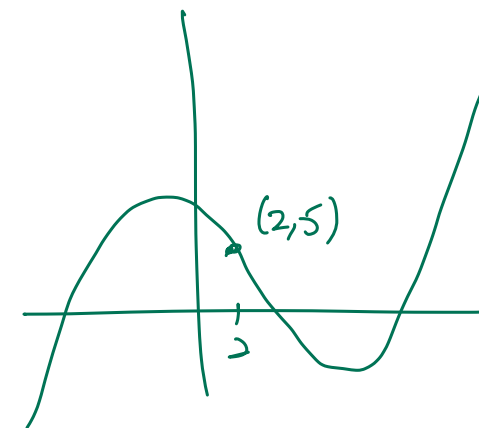
Let's do the following:

Divide $P(x) = x^3 - x^2 + 2x - 3$ by $x - 2$ and see if there is a remainder.

$$\begin{aligned} P(2) &= 2^3 - 2^2 + 2 \cdot 2 - 3 \\ &= 8 - 4 + 4 - 3 \\ &= \underline{\underline{5}} \end{aligned}$$

Just for fun, what would the value of $P(2)$ be?

$$\begin{array}{r} x^2 + x + 4 \\ x-2 \overline{) \cancel{x^3} - x^2 + 2x - 3} \\ \underline{- \cancel{x^3} - 2x^2} \\ x^2 + 2x \\ \underline{- x^2 - 2x} \\ 4x - 3 \\ \underline{- 4x - 8} \\ 5 \end{array}$$



$$x = 2 \quad \times$$

$$\underline{\underline{x - 2 = 0}}$$



Remainder theorem

If we look back at the answer to the long division of $P(x) = x^3 - x^2 + 2x - 3$ by $x - 2$ we see the following answer:

$$\frac{x^3 - x^2 + 2x - 3}{x - 2} = x^2 + x + 4 + \frac{5}{x - 2}$$

Now, when we multiply both sides by $x - 2$ look what we get

$$x^3 - x^2 + 2x - 3 = (x - 2)(x^2 + x + 4) + 5$$

This is a special form and can be used to find the remainder really easily!

$x - 2 = 0$
 $x = \underline{2}$

$x^3 - x^2 + 2x - 3 = (x - 2)(x^2 + x + 4) + 5$

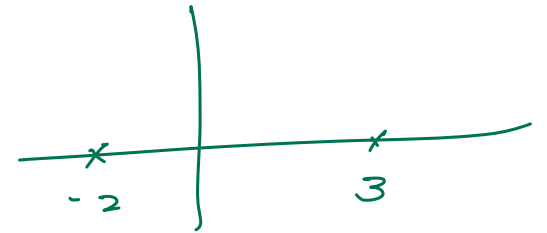
Divisor Quotient Remainder

What value of x would we need to put into the equation of the right to be left with only the 5?

$$(x + 2)(x - 3) = 0$$

$$\therefore x + 2 = 0 \quad \text{or} \quad x - 3 = 0$$

$$\underline{x = -2} \qquad \underline{x = 3}$$



Putting it another way

Because we need to get more used to Mathematical notation ... we can write this more generally

$$P(x) = (x - a)Q(x) + R$$

Which means that if we let $x = a$ we can find the value of R straight away!

But what is the value of 'a' going to be?

Well, when we had the divisor $(x - 2)$ when value of x was 2.

What will it be with $x - 4$ or $x + 3$?

$$\begin{aligned}x - a &= 0 \\ \underline{\underline{x = a}} &\leftarrow\end{aligned}$$

$$\underline{\underline{P(a) = R}}$$

$$\begin{aligned}x + 3 &= 0 \\ x &= \underline{\underline{-3}}\end{aligned}$$

$$\downarrow \\ (x - 4)$$

$$\begin{aligned}x - 4 &= 0 \\ x &= \underline{\underline{4}}\end{aligned}$$



Example: Using the remainder theorem

Find the remainder when $P(x) = x^3 - 5x^2 - x + 4$ is divided by:

a $x - 2$

b $x + 1$

$$x - 2 = 0$$

$$x = 2$$

$$P(x) = x^3 - 5x^2 - x + 4$$

$$P(2) = 2^3 - 5 \cdot 2^2 - 2 + 4$$

$$= 8 - 20 - 2 + 4$$

$$= -10$$

$$(2, -10)$$

$$x + 1 = 0$$

$$x = -1$$

$$P(-1) = (-1)^3 - 5(-1)^2 - (-1) + 4$$

$$= -1 - 5 + 1 + 4$$

$$= -1$$



Example: Finding a linear factor

Decide whether each of the following is a factor of $P(x) = x^3 + x^2 - 3x - 6$.

a $x + 1$

X

b $x - 2$

X

$(x-2)(\text{quad})$

Remember: A factor will have a remainder of zero

$$a. \quad x + 1 = 0$$

$$x = -1$$

$$P(-1) = (-1)^3 + (-1)^2 - 3(-1) - 6$$

$$= -1 + 1 + 3 - 6$$

$$= -3$$

$$(-1, 3)$$

$$x - 2 = 0$$

$$x = 2$$

$$P(2) = 2^3 + 2^2 - 3(2) - 6$$

$$= 8 + 4 - 6 - 6$$

$$= 12 - 6 - 6$$

$$= 0$$



Example: Applying the remainder theorem

Find the value of k such that $(x^3 - x^2 + 2x + k) \div (x - 1)$ has a remainder of 5.

$$p(x) = x^3 - x^2 + 2x + k$$

$$y = x^3 - x^2 + 2x + k$$

$$5 = 1^3 - 1^2 + 2 + k$$

$$5 = \cancel{1} - \cancel{1} + 2 + k$$

$$5 = 2 + k$$

$$k = \underline{\underline{3}}$$

$$x - 1 = 0$$

$$\underline{\underline{x = 1}}$$

$$\underline{\underline{y = 5}}$$

$(1, 5)$

This is an example of doing things backwards. You have been given the remainder and will have to work out what this means!



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