



# Dominance matrices

Year 12 General Maths  
Units 3 and 4

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## Learning Objectives

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By the end of the lesson, I would hope that you have an understanding and be able to apply to questions the following concepts:

- To be able to construct and interpret dominance matrices.



## Recap

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This is the last part of this section of the course before we move on to looking at how we can apply all the learnings we have gained in this section of the course.

It's really important to make sure you understand how to find the inverses of matrices (and why you might need to do this) and how to use matrices to model communication networks etc.



## What is a Dominance Matrix?

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Sport.

I don't get it.

I've learned most of my sporting knowledge from watching Ted Lasso.

But I know that lots of people like sports and so, matrices come in really handy to help us model wins and losses when teams play each other.

Dominance is basically a way of ranking the performance of something within set parameters. For example, in a round robin where teams play each other, it would be nice to know which team was the most dominant (which doesn't necessarily mean they won the most matches).

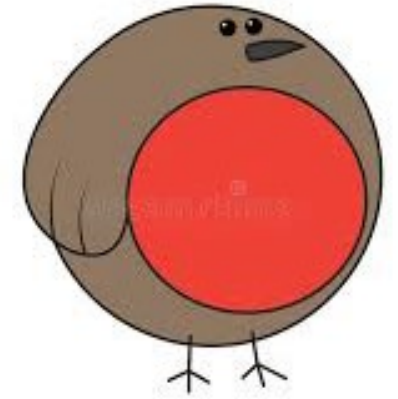
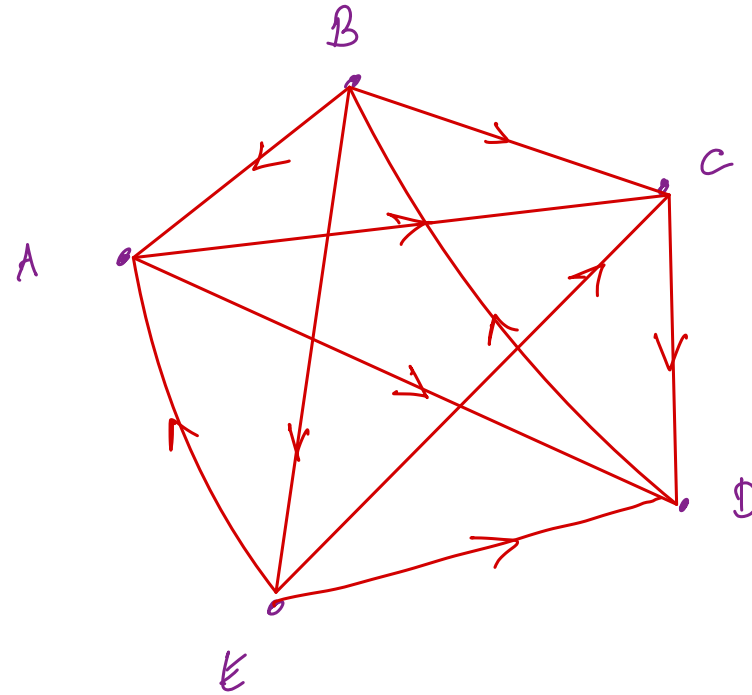


## Creating a diagram to model a round robin tournament

Let's assume that we have five players – Anna, Birgit, Cas, Di and Emma – who have played in a round-robin tournament<sup>3</sup> of tennis to see who was the dominant (best) player.

The results were as follows:

- Anna defeated Cas and Di
- Birgit defeated Anna, Cas and Emma
- Cas defeated Di
- Di defeated Birgit
- Emma defeated Anna, Cas and Di.

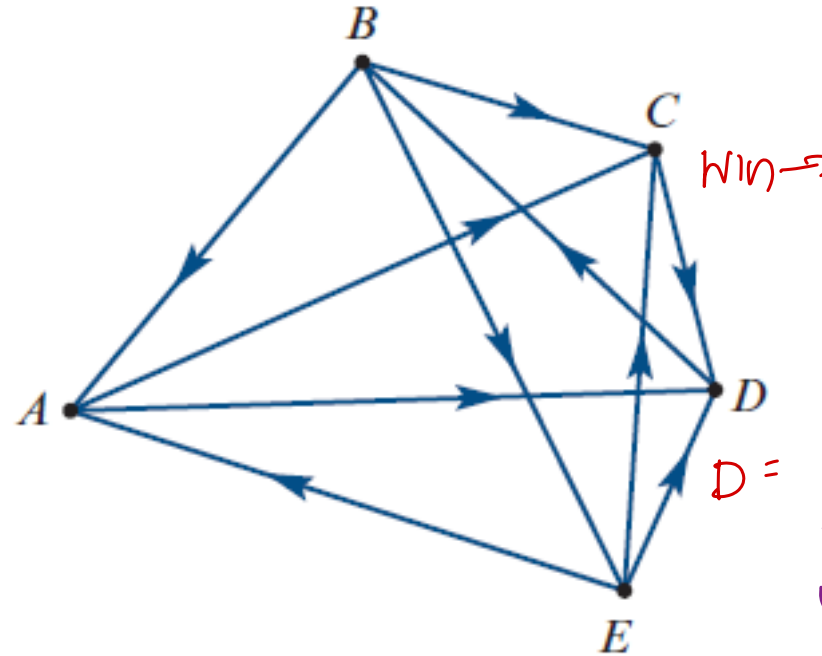


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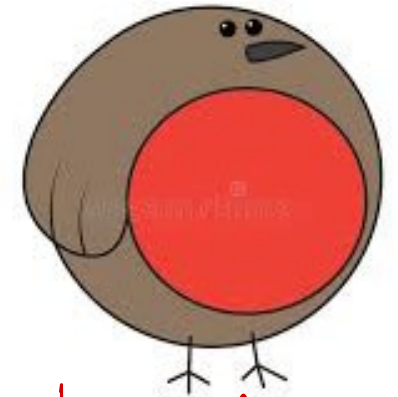
- Anna defeated Cas and Di
- Birgit defeated Anna, Cas and Emma
- Cas defeated Di
- Di defeated Birgit
- Emma defeated Anna, Cas and Di.



loses ↓

|   | A | B | C | D | E |
|---|---|---|---|---|---|
| A |   |   | 0 | 1 | 0 |
| B | 1 |   | 0 |   |   |
| C | 0 |   |   | 0 |   |
| D | 0 | 1 |   |   |   |
| E | 1 |   |   | 0 | 0 |

**Note** the directions of the arrows are pretty important and give context. The arrow points away from the winner.

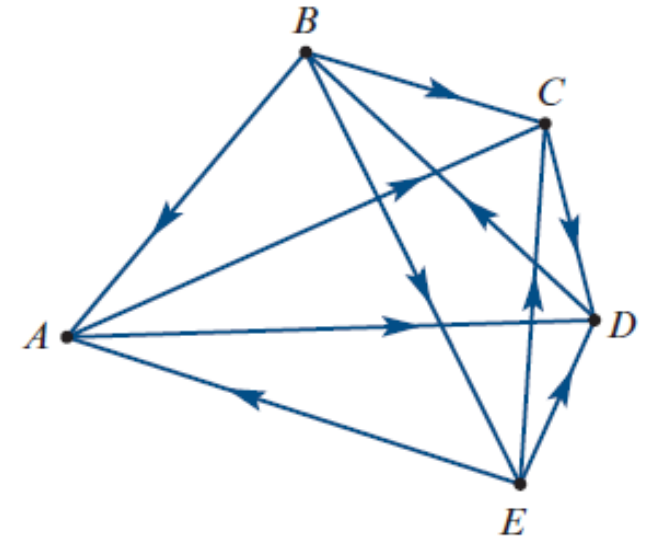


## Modelling the diagram using a matrix

Let's assume that we have five players – Anna, Birgit, Cas, Di and Emma – who have played in a round-robin tournament<sup>3</sup> of tennis to see who was the dominant (best) player.

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- Birgit defeated Anna, Cas and Emma
- Cas defeated Di
- Di defeated Birgit
- Emma defeated Anna, Cas and Di.



**Note** the directions of the arrows are pretty important and give context. The arrow points away from the winner.

$$D = \begin{array}{c|ccccc} & A & B & C & D & E & \text{Dominance} \\ \hline A & 0 & 0 & 1 & 1 & 0 & 2 \\ B & 1 & 0 & 1 & 0 & 1 & 3 \\ C & 0 & 0 & 0 & 1 & 0 & 1 \\ D & 0 & 1 & 0 & 0 & 0 & 1 \\ E & 1 & 0 & 1 & 1 & 0 & 3 \end{array}$$



# Two step dominances

This is probably one of the most important parts of this chapter and one I see assessed all the time.

A two-step dominance basically looks at who beat who.

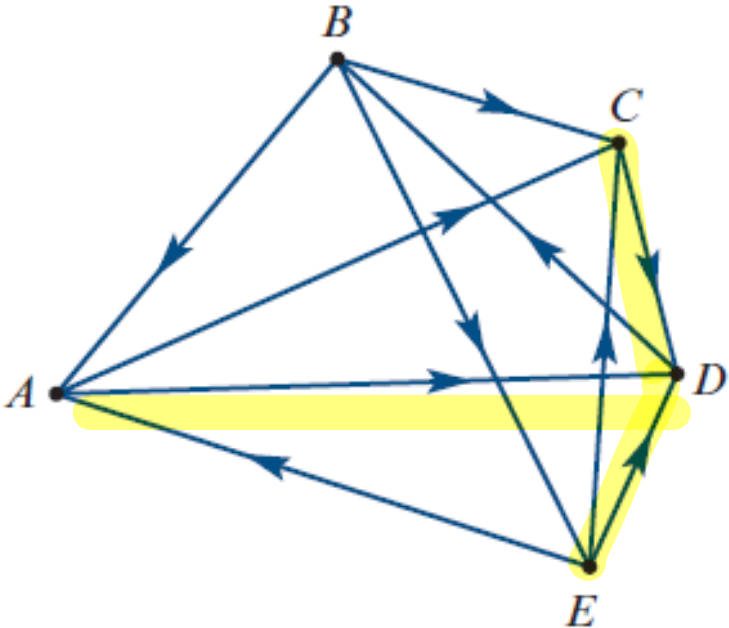
Much like we did the last time, we can work this out using the CAS. Then we can analyse what each of the numbers means.

$D =$

|   | A | B | C | D | E | Dominance |
|---|---|---|---|---|---|-----------|
| A | 0 | 0 | 1 | 1 | 0 | 2         |
| B | 1 | 0 | 1 | 0 | 1 | 3         |
| C | 0 | 0 | 0 | 1 | 0 | 1         |
| D | 0 | 1 | 0 | 0 | 0 | 1         |
| E | 1 | 0 | 1 | 1 | 0 | 3         |

$D^2 =$

|   | A | B | C | D | E | Dominance |
|---|---|---|---|---|---|-----------|
| A | 0 | 1 | 0 | 1 | 0 | 2         |
| B | 1 | 0 | 2 | 3 | 0 | 6         |
| C | 0 | 1 | 0 | 0 | 0 | 1         |
| D | 1 | 0 | 1 | 0 | 1 | 3         |
| E | 0 | 1 | 1 | 2 | 0 | 4         |



# Total dominance

If we wanted to find out the most dominant team, we would then add the one step and two-step dominances. This would give us a ranking of best to worst team.

$D =$ 

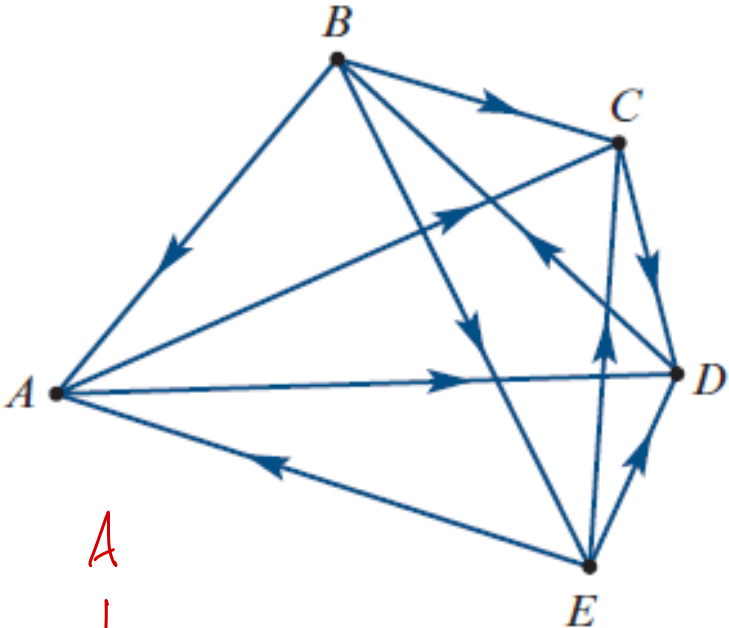
|   | A | B | C | D | E | Dominance |
|---|---|---|---|---|---|-----------|
| A | 0 | 0 | 1 | 1 | 0 | 2         |
| B | 1 | 0 | 1 | 0 | 1 | 3         |
| C | 0 | 0 | 0 | 1 | 0 | 1         |
| D | 0 | 1 | 0 | 0 | 0 | 1         |
| E | 1 | 0 | 1 | 1 | 0 | 3         |

$D^2 =$ 

|   | A | B | C | D | E | Dominance |
|---|---|---|---|---|---|-----------|
| A | 0 | 1 | 0 | 1 | 0 | 2         |
| B | 1 | 0 | 2 | 3 | 0 | 6         |
| C | 0 | 1 | 0 | 0 | 0 | 1         |
| D | 1 | 0 | 1 | 0 | 1 | 3         |
| E | 0 | 1 | 1 | 2 | 0 | 4         |

$T = D + D^2 =$ 

|   | A | B | C | D | E | Total |
|---|---|---|---|---|---|-------|
| A | 0 | 1 | 1 | 2 | 0 | 4     |
| B | 2 | 0 | 3 | 3 | 1 | 9     |
| C | 0 | 1 | 0 | 1 | 0 | 2     |
| D | 1 | 1 | 1 | 0 | 1 | 4     |
| E | 1 | 1 | 2 | 3 | 0 | 7     |



A  
↓  
B  
↓  
C

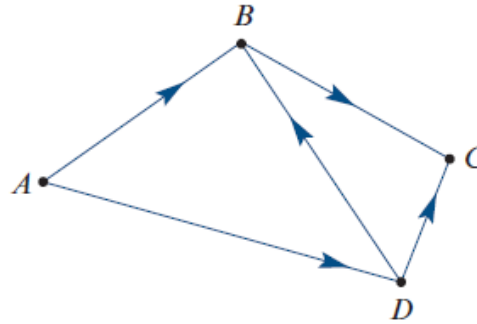


## Example: Dominance Matrices

Four people,  $A$ ,  $B$ ,  $C$  and  $D$ , have been asked to form a committee to decide on the location of a new toxic waste dump.

From previous experience, it is known that:

- $A$  influences the decisions of  $B$  and  $D$
  - $B$  influences the decisions of  $C$
  - $C$  influences the decisions of no one
  - $D$  influences the decisions of  $C$  and  $B$ .
- a** Use the graph to construct a dominance matrix that takes into account both one-step and two-step dominances.
- b** From this matrix, determine who is the most influential person on the committee.



$D = \text{influences}$

*influences*

|     | $A$ | $B$ | $C$ | $D$ |
|-----|-----|-----|-----|-----|
| $A$ | 0   | 1   | 0   | 1   |
| $B$ | 0   | 0   | 1   | 0   |
| $C$ | 0   | 0   | 0   | 0   |
| $D$ | 0   | 1   | 1   | 0   |



$$T = D + D^2$$
$$=$$

$D^2$



## VCAA Questions

### Question 3

Five people, India ( $I$ ), Jackson ( $J$ ), Krishna ( $K$ ), Leanne ( $L$ ) and Mustafa ( $M$ ), competed in a table tennis tournament.

Each competitor played every other competitor once only.

Each match resulted in a winner and a loser.

The matrix below shows the tournament results.

A '1' in the matrix shows that the competitor named in that row defeated the competitor named in that column.

For example, the '1' in the fourth row shows that Leanne defeated Jackson.

A '0' in the matrix shows that the competitor named in that row lost to the competitor named in that column.

There is an error in the matrix. The winner of one of the matches has been incorrectly recorded as a '0'.

This match was between

- ☒ A. India and Mustafa.
- ☐ B. India and Krishna.
- ☐ C. Krishna and Leanne.
- ☐ D. Leanne and Mustafa.
- ☐ E. Jackson and Mustafa.

|               |     | <i>loser</i> |     |     |     |     |
|---------------|-----|--------------|-----|-----|-----|-----|
|               |     | $I$          | $J$ | $K$ | $L$ | $M$ |
| <i>winner</i> | $I$ | 0            | 1   | 0   | 1   | 0   |
|               | $J$ | 0            | 0   | 1   | 0   | 1   |
|               | $K$ | 1            | 0   | 0   | 1   | 1   |
|               | $L$ | 0            | 1   | 0   | 0   | 0   |
|               | $M$ | 0            | 0   | 0   | 1   | 0   |



2018 Further Maths

Paper 1

Question 3

Correct: 78%

## VCAA Questions

Five competitors, Andy (*A*), Brie (*B*), Cleo (*C*), Della (*D*) and Eddie (*E*), participate in a darts tournament. Each competitor plays each of the other competitors once only, and each match results in a winner and a loser.

The matrix below shows the results of this darts tournament.

There are still two matches that need to be played.

A '1' in the matrix shows that the competitor named in that row defeated the competitor named in that column.

For example, the '1' in row 2, column 3 shows that Brie defeated Cleo.

A '...' in the matrix shows that the competitor named in that row has not yet played the competitor named in that column.

Which player, by winning their remaining match, will ensure that they are ranked first by the sum of their one-step and two-step dominances?

- A. Andy
- B. Brie
- C. Cleo
- D. Della
- E. Eddie

|               |          | <i>loser</i> |          |          |          |          |
|---------------|----------|--------------|----------|----------|----------|----------|
|               |          | <i>A</i>     | <i>B</i> | <i>C</i> | <i>D</i> | <i>E</i> |
| <i>winner</i> | <i>A</i> | 0            | !        | 0        | 1        | 0        |
|               | <i>B</i> | ?            | 0        | 1        | 0        | 1        |
|               | <i>C</i> | 1            | 0        | 0        | !        | 1        |
|               | <i>D</i> | 0            | 1        | ?        | 0        | 0        |
|               | <i>E</i> | 1            | 0        | 0        | 1        | 0        |

2020 Further Maths  
Paper 1  
Question 9  
Correct: 63%



# VCAA Questions

Five staff members in Elena’s office played a round-robin video game tournament, where each employee played each of the other employees once. In each game there was a winner and a loser.

A table of their one-step and two-step dominances was prepared to summarise the results.

Consider the results matrix shown below.

A ‘1’ in this matrix shows that the player named in that row defeated the player named in that column.

A ‘0’ in this matrix shows that the player named in that row lost to the player named in that column.

Use all of the information provided to complete the results matrix.

| Staff member         | One-step dominance | Two-step dominance |
|----------------------|--------------------|--------------------|
| Ike ( <i>I</i> )     | 3                  | 5                  |
| Joelene ( <i>J</i> ) | 3                  | 4                  |
| Katie ( <i>K</i> )   | 1                  | 1                  |
| Leslie ( <i>L</i> )  | 1                  | 2                  |
| Mikki ( <i>M</i> )   | 2                  | 4                  |

|               |          |              |          |          |          |          |
|---------------|----------|--------------|----------|----------|----------|----------|
|               |          | <i>loser</i> |          |          |          |          |
|               |          | <i>I</i>     | <i>J</i> | <i>K</i> | <i>L</i> | <i>M</i> |
| <i>winner</i> | <i>I</i> | 0            | 1        | 1        | 1        | 0        |
|               | <i>J</i> | 0            | 0        | 1        | 1        | 1        |
|               | <i>K</i> | 0            | 0        | 0        | 1        | 0        |
|               | <i>L</i> | 0            | 0        | 0        | 0        | 1        |
|               | <i>M</i> | 1            | 0        | 1        | 0        | 0        |

2021 Further Maths  
Paper 2  
Question 4  
Correct: 10%

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