



Binary, permutation and communication matrices

Year 12 General Maths
Units 3 and 4

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Learning Objectives

By the end of the lesson, I would hope that you have an understanding and be able to apply to questions the following concepts:

- To be able to identify and work with binary matrices, permutation matrices and communication matrices.



Recap

Having worked on the basics, we now have to start applying the work on matrices to real world problems. Matrices can be used to solve real world practical problems. They can show communication paths, dominance in sporting leagues and so much more!

This is going to build on the work we have been doing in previous lessons relating to:

- Adding matrices
- Subtracting matrices
- Multiplying matrices
- Inverse matrices etc



What is a Binary Matrix?

Binary is a number system which uses only ones and zeros.

Everything you store on your phone, computer, iPad etc will be stored as a sequence of ones and zeros. Your computer stores them in a certain pattern which it can read make and turn into things which makes sense to us.

So, a Binary matrix can be any matrix (of any size) which is populated only by ones and zeros.

$$\begin{bmatrix} 1 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$



What is a Permutation Matrix?

I remember doing permutations and combinations when I was back in school in the UK. And, to be honest, never really understood them then ... but now I do! If only I could go back and do my school life all over again!!!

When we **permute** something we are simply changing the order.

So, an permutation of DARREN would be NERRAD.

There are LOTS of ways of permutating my name but what if I wanted to permute them in a certain order? Well, we can use a matrix.

DARREN

NERRAD

RADREN



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Example: Permutation Matrix

The language in Mathematics is the thing which tend to trip people up.

So, let's just get the following out of the way before we get into the example:

- **Pre:** means before
- **Post:** means after

pre :

post :

A.B

B.A



Example: Permutation Matrix

X is the column matrix $X = \begin{bmatrix} T \\ A \\ R \end{bmatrix}$. P is the permutation matrix $P = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$.

a Show that:

i pre-multiplying² X by P changes the matrix X to the matrix $Y = \begin{bmatrix} R \\ A \\ T \end{bmatrix}$

ii pre-multiplying X by P^2 leaves the matrix X unchanged.

b What can be deduced about P^2 from the result in **a ii**?

Note: A CAS can make this so much easier. Too many people just don't take the time to learn how to do this on their CAS

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} T \\ A \\ R \end{bmatrix} = \begin{bmatrix} R \\ A \\ T \end{bmatrix}$$

$P \cdot X$

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} R \\ A \\ T \end{bmatrix} = \begin{bmatrix} \\ \\ \end{bmatrix}$$

$P^2 \cdot X$



Understanding what the 1's actually do in a Permutation Matrix

Let's look at why the 1's and 0's move things by looking back at P:

$$P = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\begin{array}{l} \text{1st row} \\ \text{2nd row} \\ \text{3rd row} \end{array} \begin{array}{c} \text{1st} \quad \text{2nd} \quad \text{3rd} \\ \left[\begin{array}{ccc} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{array} \right] \left[\begin{array}{c} T \\ A \\ R \end{array} \right] = \left[\begin{array}{c} R \\ A \\ T \end{array} \right] \end{array}$$

Note: There cannot be more than one 1 in a row or column.



Example: Permutation Matrix

Find a permutation matrix that takes the column matrix $\begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix}$ to the column matrix $\begin{bmatrix} D \\ C \\ B \\ A \end{bmatrix}$

$$\begin{array}{l} \text{1st row} \\ \text{2nd row} \\ \text{3rd row} \\ \text{4th row} \end{array} \begin{array}{cccc} \text{1st} & \text{2nd} & \text{3rd} & \text{4th} \\ \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \end{array} \begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix} = \begin{bmatrix} D \\ C \\ B \\ A \end{bmatrix}$$

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Interesting fact: Permutation Matrix

Let's look at a Permutation Matrix and it's inverse and see what happens:

$p :=$	$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$	p^T	$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$
p^{-1}	$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$		

p^T	$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$
p^{-1}	$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$

Basically, we notice that the inverse is the same as the transpose of the original matrix.

Note: not too sure why this is important but it's in the new text book!



Communication matrices

Once upon a time, telephone networks were connected by wires.

These wires went to a central exchange and you would ring the “operator” and ask to be connected to another person. She would literally plug two people together.

Operator allowed two people to communicate. Taking this one step further, what if the operator spoke French and English. I speak English (badly) and I wanted to talk to someone in French. She would act as a translator.

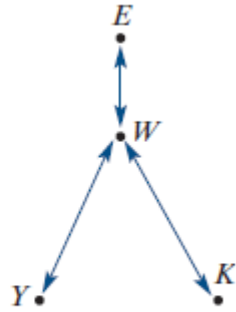
Let’s look at how Matrices can be used to model communication networks.



Communication matrices

Eva, Wong, Yumi and Kim are students who are staying in a backpacker's hostel. Because they speak different languages they can have problems communicating. The situation they have to deal with is that:

- Eva speaks English only
- Yumi speaks Japanese only
- Kim speaks Korean only
- Wong speaks English, Japanese and Korean.



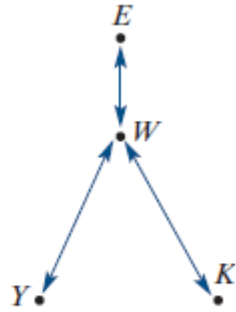
Note: The order of the rows and columns in a communication matrix is **really important**.



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One step communication link: A direct connection between two people.

Two step communication link: A communication link which takes two steps to complete the message.

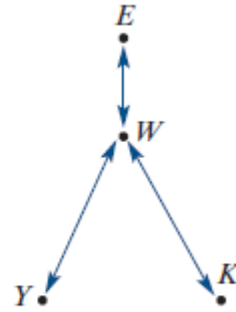
		Receiver			
		E	W	Y	K
Speaker	E	0	1	0	0
	W	1	0	1	1
	Y	0	1	0	0
	K	0	1	0	0



Two step communication matrices

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receiver.

sender

	<i>E</i>	<i>W</i>	<i>Y</i>	<i>K</i>	
	1	0	1	1	<i>E</i>
	0	3	0	0	<i>W</i>
	1	0	1	1	<i>Y</i>
	1	0	1	1	<i>K</i>

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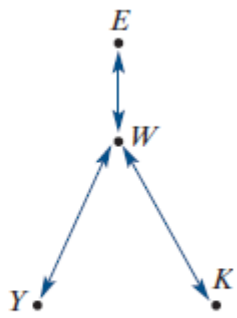
This can be done by hand and using a CAS!



Two step communication matrices: Using a CAS

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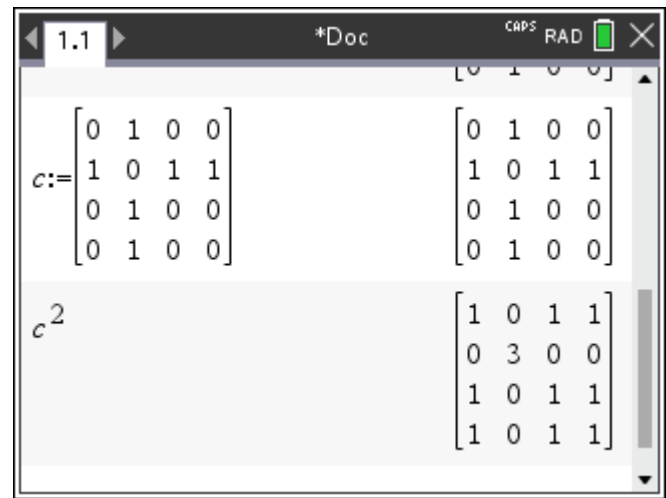
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		Receiver			
		E	W	Y	K
Speaker	E	0	1	0	0
	W	1	0	1	1
	Y	0	1	0	0
	K	0	1	0	0



Redundant communication links

If the sender and receiver are the same person we call this a redundant communication link.

It might be helpful to remove these leading diagonal numbers when being asked to find two-step or three-step communication links.



VCAA Questions

Question 4

The communication matrix below shows the communication links between five people: Steph (S), Tran (T), Ursula (U), Vinh (V) and Wanda (W).

	receiver				
	S	T	U	V	W
S	0	1	1	0	1
T	0	0	0	1	1
U	0	1	0	1	0
V	0	0	1	0	0
W	1	0	0	1	0

U-T $\swarrow \searrow$ V
U-V

~~U-T-V~~
U-T-W-S
~~U-V~~

In this matrix:

- the '1' in row S, column T indicates that Steph can communicate directly with Tran
- the '0' in row V, column W indicates that Vinh cannot communicate directly with Wanda.

Ursula needs to communicate with Steph.

The sequence of communication links that will successfully allow Ursula to communicate with Steph is

- A. U-T-S
- B. U-W-S
- C. U-T-W-S**
- D. U-V-T-S
- E. U-W-T-S

2022 Further Maths
Paper 1
Question 4
Correct: 87%



VCAA Questions

Question 7

Matrix K is a permutation matrix.

$$K = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Matrix M is a column matrix that is multiplied once by matrix K to obtain matrix P .

When matrix M is multiplied by matrix K , the element m_{31} moves to element

A. p_{11}

B. p_{21}

C. p_{31}

D. p_{41}

E. p_{51}

Handwritten solution showing the multiplication of matrix K and column matrix M to obtain matrix P .

Matrix K is shown with columns labeled 1 to 5. The element m_{31} is highlighted in the third row, first column of M .

The resulting matrix P is shown with elements $p_{11}, p_{21}, p_{31}, p_{41}, p_{51}$.

Handwritten notes on the left indicate the row indices for the elements in M :

- 1st row: m_{11}
- 2nd row: m_{21}
- 3rd row: m_{31} (highlighted)
- 4th row: m_{41}
- 5th row: m_{51}

2022 Further Maths

Paper 1

Question 7

Correct: 42%



VCAA Questions

Question 2 (3 marks)

The main computer system in Elena's office has broken down.

The five staff members, Alex (A), Brie (B), Chai (C), Dex (D) and Elena (E), are having problems sending information to each other.

Matrix M below shows the available communication links between the staff members.

$$M = \begin{matrix} & \begin{matrix} \text{receiver} \\ A & B & C & D & E \end{matrix} \\ \begin{matrix} \text{sender} \\ A \\ B \\ C \\ D \\ E \end{matrix} & \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{matrix}$$

In this matrix:

- the '1' in row A , column B indicates that Alex can send information to Brie
- the '0' in row D , column C indicates that Dex cannot send information to Chai.

- c. Matrix M^2 below is the square of matrix M and shows the number of two-step communication links between each pair of staff members.

$$M^2 = \begin{matrix} & \begin{matrix} \text{receiver} \\ A & B & C & D & E \end{matrix} \\ \begin{matrix} \text{sender} \\ A \\ B \\ C \\ D \\ E \end{matrix} & \begin{bmatrix} 0 & 0 & 1 & 2 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \end{matrix}$$

Only one pair of individuals has two different two-step communication links.

List each two-step communication link for this pair.

1 mark

Alex - Brie - Dex
Alex - Elena - Dex



2022 Further Maths

Paper 2

Question 2

Correct: 21%

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