The parabola

Year 9 Mathematics
Mainstream

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Learning Objectives

By the end of the lesson I hope that you understand and can apply the following to a range of questions from the Year 9 Mathematics course.

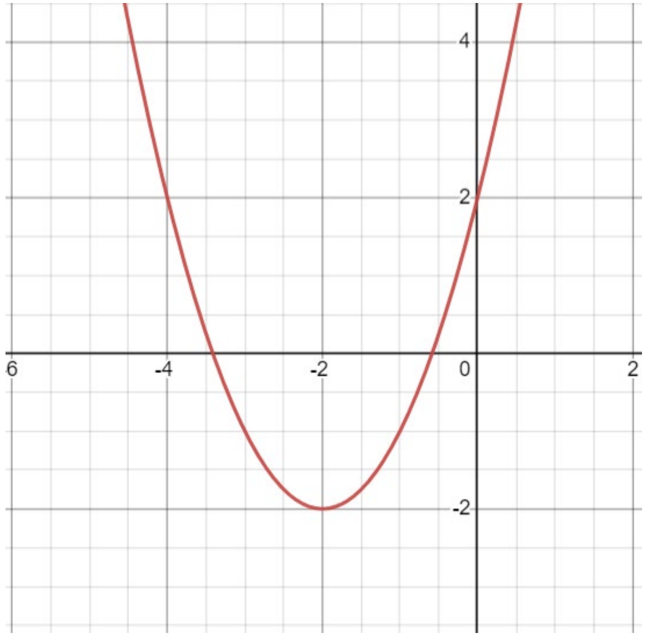
- Understand that a quadratic equation when drawn is a parabola (or is parabolic in shape).
- Understand how to draw a parabola
- Understand what it means to find the solutions to a quadratic and use it to sketch a parabola
- Understand what an axis of symmetry is
- Know what it means by the turning point and find it

RECAP

We have, in previous lessons been looking at the ways we can use to solve quadratic equations. In each question we have been reminded that we are, when solving, simply finding the point (or points) where a parabola cross the x-axis.

Seems rather odd them to only now be looking at what a parabola is! But … such is life!

This lesson will look more at the parabola and show us how we can use the solutions to help us find the turning points (minimum or maximum) and how to find the y-axis intercept.



Remember:
Quadratics can have two, one or zero solutions where a solution is where the graph crosses or touches the x-axis.

A parabola

A parabola is basically a ‘U’ or an ‘n’ shape.

It is symmetrical.

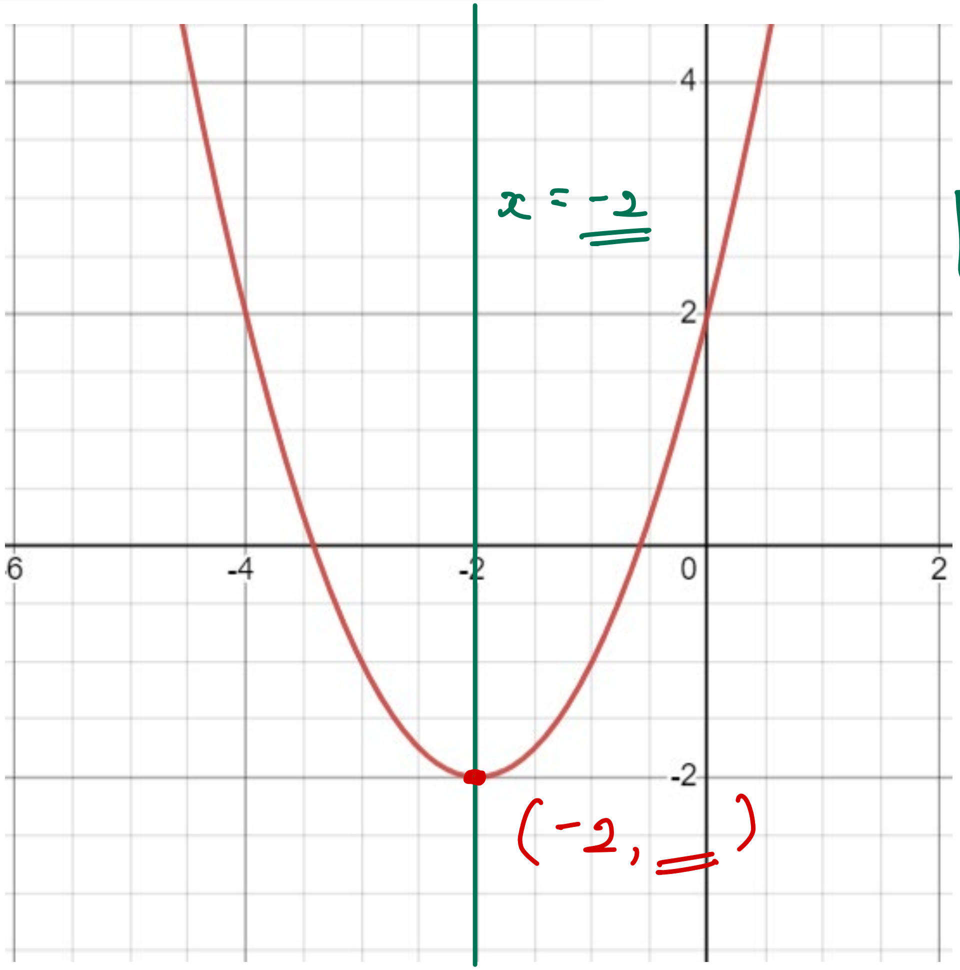
We can draw the line of symmetry vertically through the minimum or maximum point. **This is called the axis of symmetry**.

We can find “solutions” by solving quadratic equations using the methods we have already learned.

We know that the x-coordinate of the turning point will fall exactly between the two solutions (when there are two solutions!) or between two points which lie on the curve with the same y-coordinate.

minimum

maximum

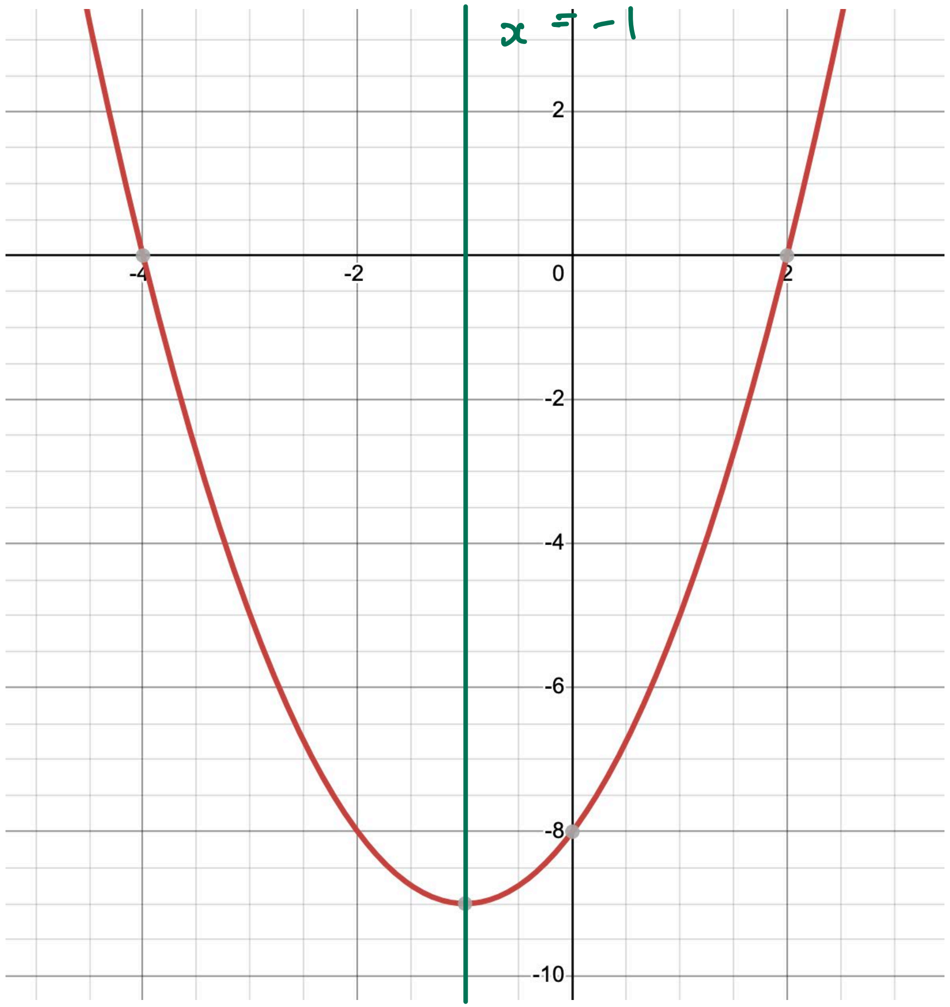


$\boxed{a}x^2$

$-ax^2$

Examples of parabola’s

Here are some examples which show how we can find the turning point of a parabola



x int $= (-4, 0)$

$= (2, 0)$

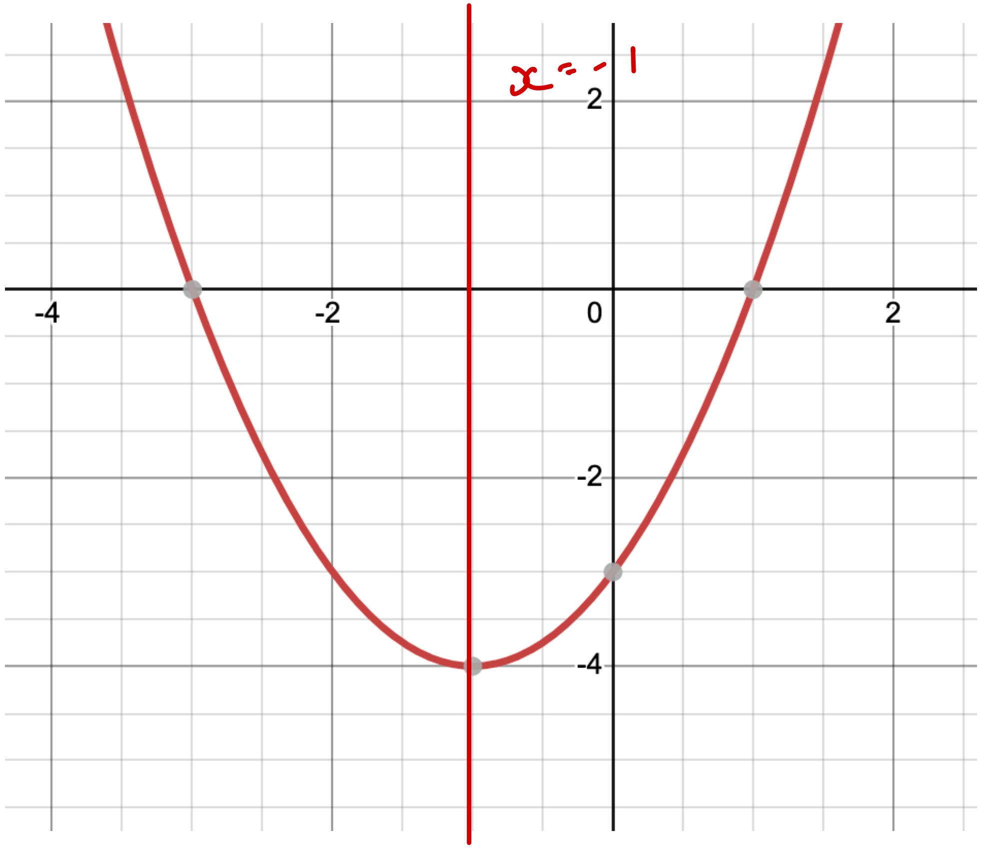
y int $= (0, -8)$

$x = -1$

min $= (-1, -9)$

Examples of parabola’s

Here are some examples which show how we can find the turning point of a parabola



x int $= (-3, 0)$

$= (1, 0)$

y int $= (0, -3)$

$x = -1$

min $= (-1, -4)$

Examples of parabola’s

Here are some examples which show how we can find the turning point of a parabola

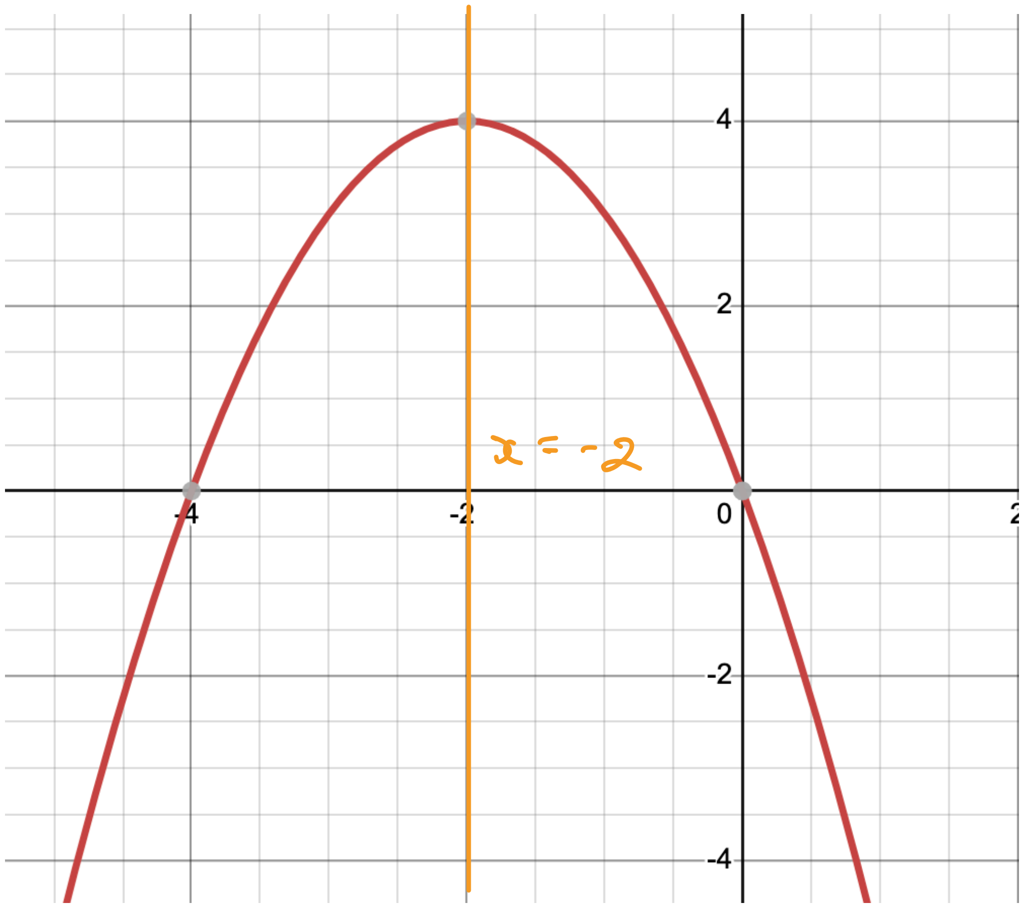
x int $= (-4, 0)$

$= (0, 0)$

y int $= (0, 0)$

$x = -2$

max $= (-2, 4)$



Finding the equation of a parabola from the two intercepts

We have already looked at how we can do this in a previous lesson but it's important to recap it as, with the equation, we can find the y-values of the turning points and the intercept.

Using the graph on the right we can see the intercepts are -4 and 2.

This means we can write the factorised form of the equation as:

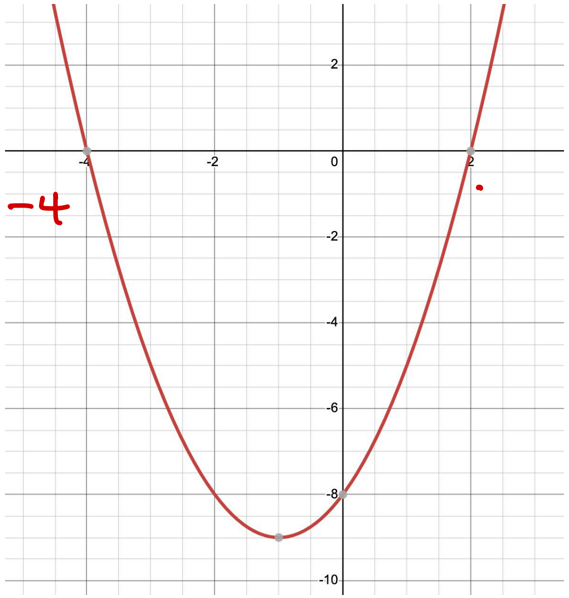
$$y = (x + 4)(x - 2)$$

This allows us to expand the brackets and find the quadratic equation.

$$y = (x+4)(x-2)$$

$$= x^2 - 2x + 4x - 8$$

$$= x^2 + \underline{\underline{2x}} - 8$$



-4

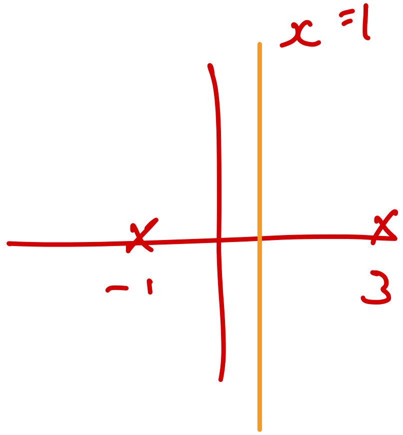
y int

$x = 0$

Remember: The y-axis intercept is found when the x-value is equal to zero.

Finding the equation of a parabola from the two intercepts

Find the turning point and y-axis intercept if you are told the solutions of a quadratic equation are (-1, 0) and (3, 0)



$2x - 2$

$y = (x+1)(x-3)$

$= x^2 - 3x + x - 3$

$= x^2 - 2x - 3$

y int

$\therefore$ y int $= (0, -3)$

tp = min = $(1, -4)$

Warning Will Robinson …

Sadly, we have a bit of a problem.

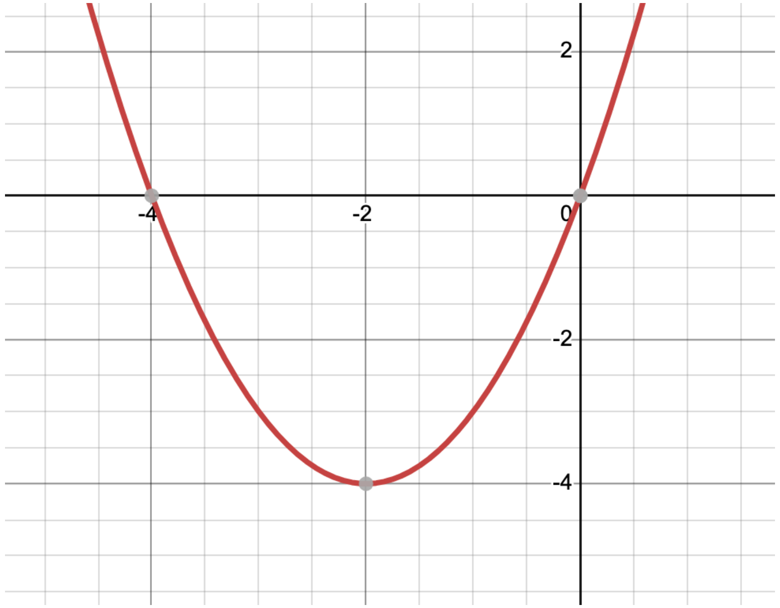
When we look at the following two graphs, we see that they have the same x-axis intercepts, but they have very different turning points!

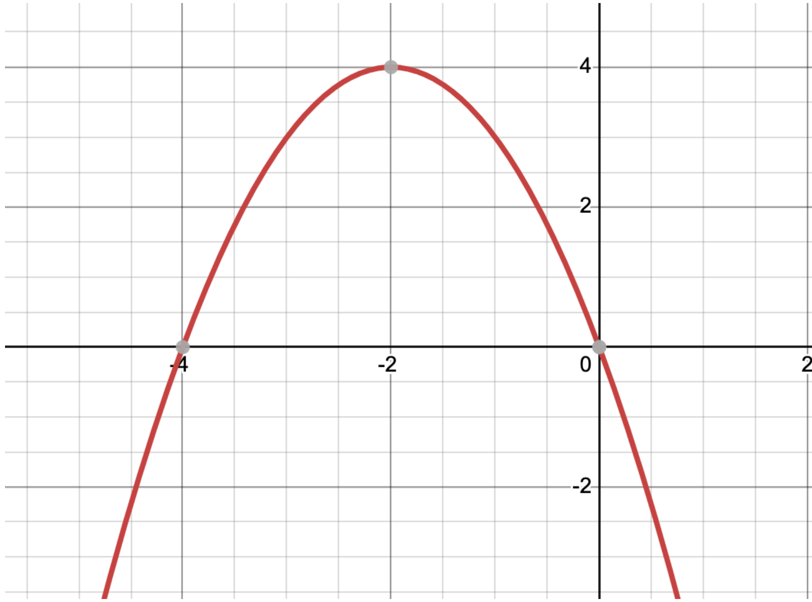
This is because the equation of the top quadratic is:

$$y = x^2 + 4x$$

And the second graph is:

$$y = -x^2 - 4x$$





To find the actual equation is beyond the scope of this lesson, but there are ways!

Example: Identifying the features of a parabola

For this graph state the:

- equation of the axis of symmetry
- type of turning point
- coordinates of the turning point
- x-intercepts
- y-intercept.

$x = -3$

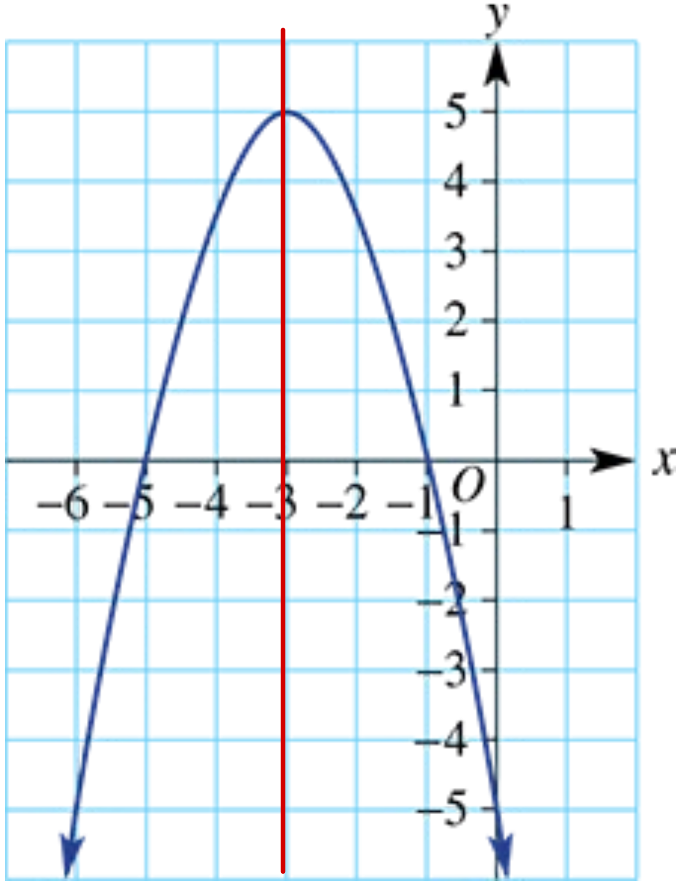
Maximum

$tp = (-3, 5)$

x int $= (-5, 0)$

$(-1, 0)$

y int $= (0, -5)$



Doing things, the old-fashioned way

Long before you were even a twinkle in your parents eye, I was having to do Maths the hard way. We had to plot these things!!!!

Using a table and some real sums.

It's about time you guys learned some real maths! So, you're going to do the same.

A quadratic is given by the equation $y = x^2 - 2x - 3$. Complete these tasks to discover its graphical features.

Use the rule to complete this table of values.

x	−2	−1	0	1	2	3	4
y	5	0	−3	-4	−3	0	5

$y = x^2 - 2x - 3$

1 +2 −3

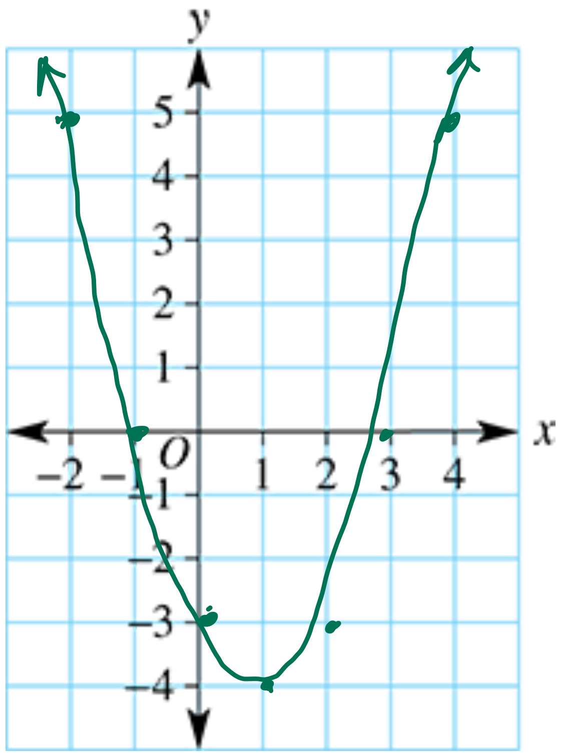
1 −2 −3 =

4 − 4 −3

9 − 6 −3

16 − 8 −3

Plot your points on a copy of the axes shown at right and join them to form a smooth curve.



Example: Plotting a parabola

$y = x^2 - 4$

Use the quadratic rule $y = x^2 - 4$ to complete these tasks.

Complete this table of values.

x	−2	−1	0	1	2	3	4
y	0	-3	-4	-3	0	5	12

$1 - 4 = -3$

$1 - 4$

Draw a set of axes using a scale that suits the numbers in your table. Then plot the points to form a parabola and state these features.

- Type of turning point
- Axis of symmetry
- Coordinates of the turning point
- The y-intercept
- The x-intercepts

minimum

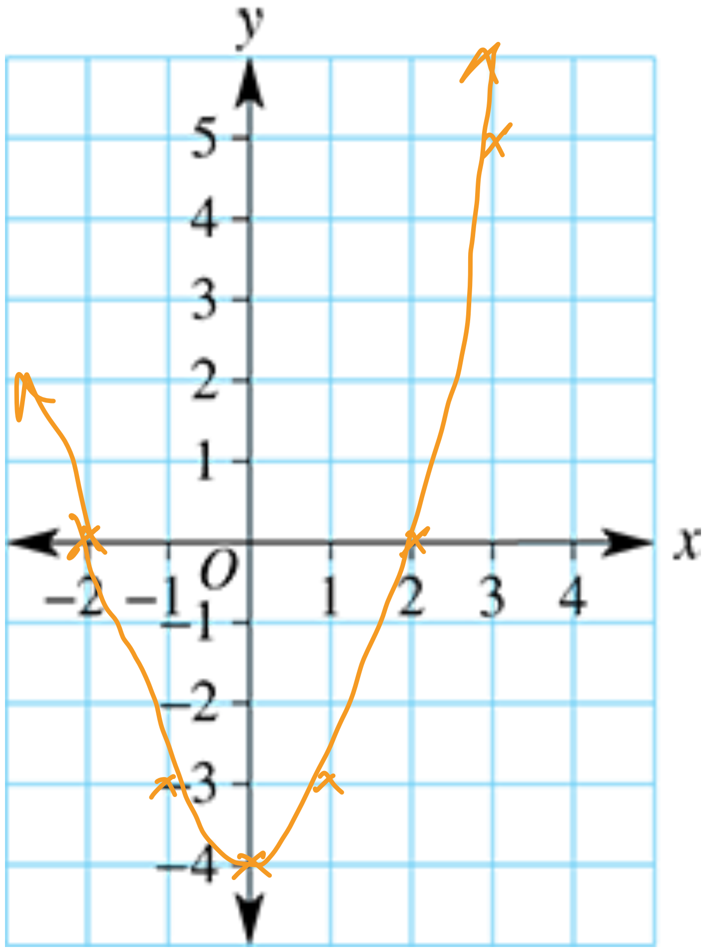
$x = 0$

$(0, -4)$

y int $= (0, -4)$

x int $= (-2, 0)$

$(2, 0)$



Questions to complete:

The questions I would like you to complete for this lesson are:

Exercise 10E The parabola
Questions: 1, 2, 5, 6, 7, 8, 11, 13

Extension: 14

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