Solving

$$x^2 + bx + c = 0$$

Year 9 Mathematics
Mainstream

Learning Objectives

By the end of the lesson I hope that you understand and can apply the following to a range of questions from the Year 9 Mathematics course.

- Know what a monic trinomial is
- Know how to factorise monic trinomials
- Know that a perfect square will only have on solution.

RECAP

We have, in the previous lesson, looked at the fact that there are patterns to the way quadratics look. If we see the pattern then we can choose the correct way to factorise is.

Last lesson we looked at quadratics which have the form:

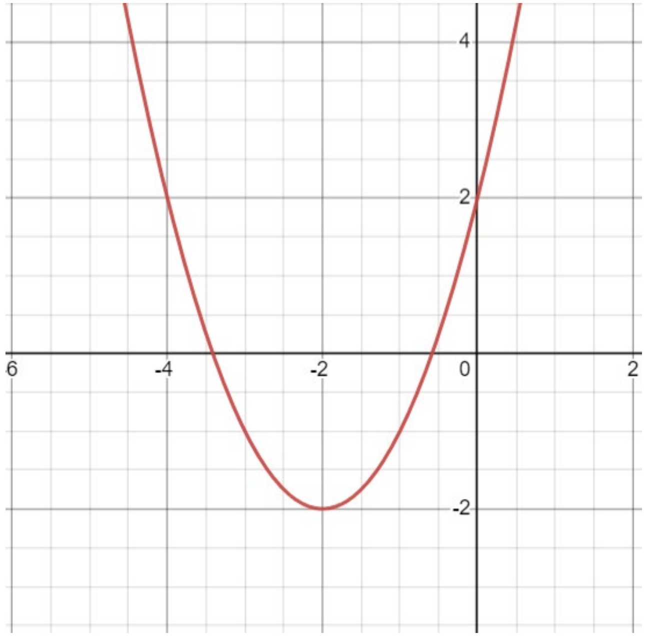
$$ax^2 = d$$

And

$$ax^2 + bx = 0$$

The first example simply needs us to square root the answer, remembering that we will have a positive and negative solutions.

The second example requires us to factorise and then use the null factor law.



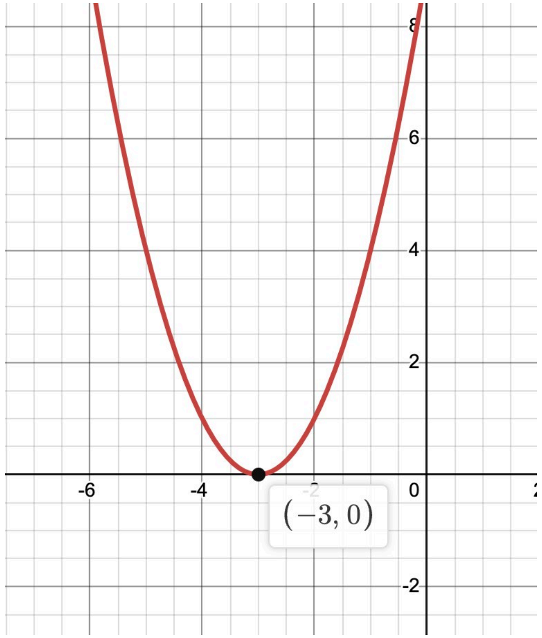
When we solve quadratics we are finding solutions. This means we are finding where the graphs crosses the x-axis, or where the y-values are zero.

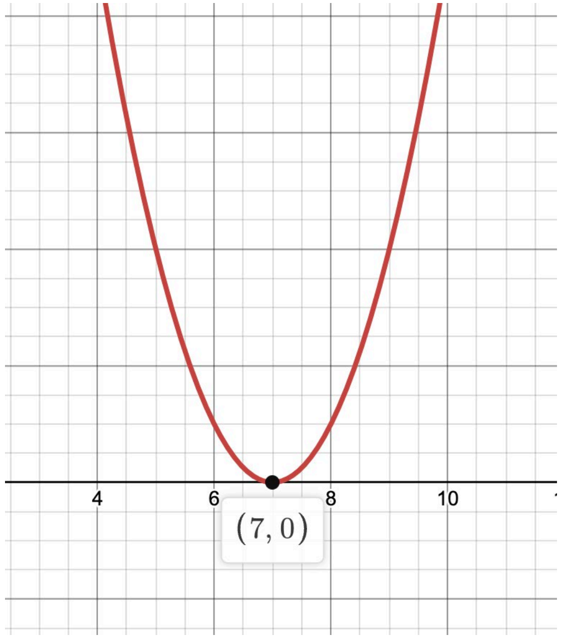
Looking at perfect squares

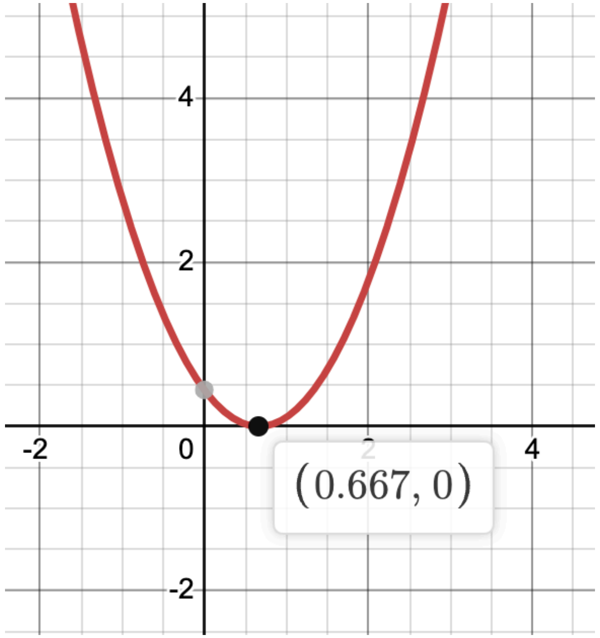
A "perfect square" is when a quadratic can be factorised to look like the following:

$$(x+3)^2 = 0$$
$$(x-7)^2 = 0$$
$$\left(x-\frac{2}{3}\right)^2 = 0$$

When we look at each of the graphs in turn. We can see something interesting about these graphs.







x^2 3^2 9^2 10^2

$(x-3)^2$ $(x+4)^2$

Understanding perfect squares

As we can see from the following, there are a number of “interesting things” to note.

$$(x+3)^2 = 0$$

1. The graph only touches the –axis once.
2. The value it touches as *appears* to bear a relationship to the number inside the brackets; it’s the same number but with the sign swapped.

When a graph touches the x-axis in only one place, we say that the solution is a **repeated root**.

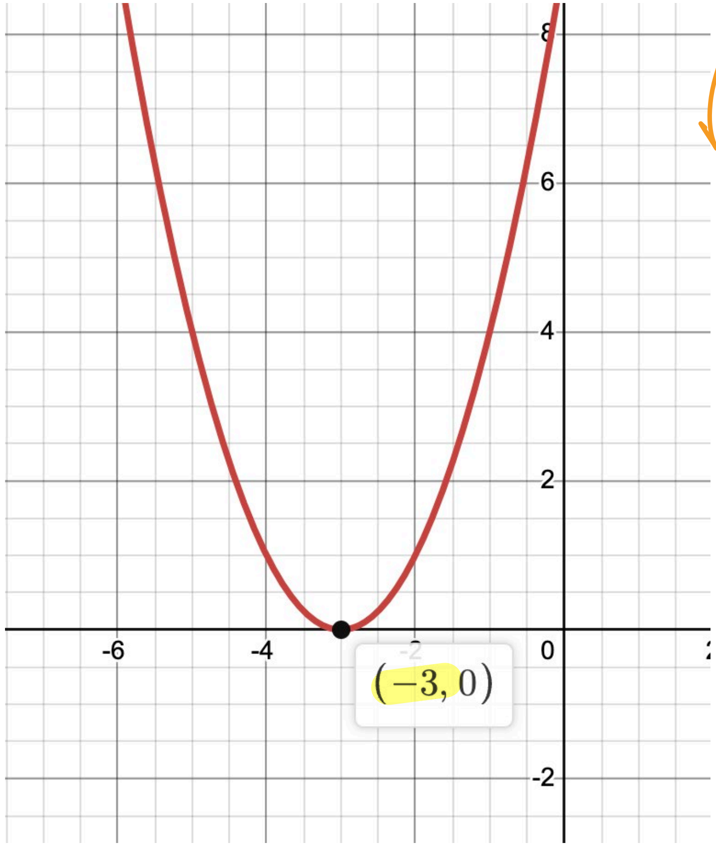
The reason is clear when we use algebra to solve.

$$(x+3)^2 = 0$$

$$(x+3)(x+3) = 0$$

$$\therefore\ x+3 = 0 \quad \text{or} \quad x+3 = 0$$

$$x = -3 \qquad x = -3$$



$$\sqrt{(x+3)^2} = \sqrt{0}$$

$$x+3 = 0$$

$$x = -3$$

Monic Trinomials

Barry has been at is again and is now calling quadratic equations trinomials.

Sigh. Just write it in your summary book and remember it for all time.

But what about the monic?

The following are examples of monic trinomials:

$$x^2 - 3x + 2$$
$$x^2 - 9 = 0$$
$$x^2 + 5x = 0$$

These are examples of non-monic trinomials:

$$3x^2 - 3x + 2$$
$$1x^2 - 9 = 0$$
$$5x^2 + 5x = 0$$

So, monic trinomials have a '1' as a coefficient for the x^2 term.

Solving monic trinomials

We can solve them using the methods we have learned earlier in the course. As that is a long time ago, we can refresh our knowledge with some examples.

It's important to remember that:

- we are looking to factorise by **grouping pairs**
- We cannot factorise three terms
- We are looking at using the null factor law
- Solutions are where the graph crosses the x-axis.

We can use the 'T' method to help us solve these types of problems.

RECAP: Using the 'T' Method

When using the T-Method you are looking at creating a 'T' on the left hand side of the page.

At the top of the 'T' you put the product of the 'a' and 'c' terms in the quadratic.

Remember: Quadratics must be written in the form $ax^2 + bx + c = 0$ for us to be able to solve them.

You are looking at finding all the factors of the number at the top of the 'T'. Two of these factors will add to make the value of 'b' in the quadratic. These are the two you use to turn **three into four** and then group.

An example will help!

Solve the quadratic equation: $x^2 + 7x + 12 = 0$

$$1x^2 + 7x + 12 = 0$$

$$x^2 + 3x + 4x + 12 = 0$$

$$x(x+3) + 4(x+3) = 0$$

$$(x+3)(x+4) = 0$$

$+12$

~~1~~	~~12~~
~~2~~	~~6~~
3	4

$\therefore x + 3 = 0$

$x = -3$

or $x + 4 = 0$

$x = -4$

Examples: Solving equations with quadratic trinomials

Solve $x^2 - 2x - 8 = 0$

$$x^2 - 2x - 8 = 0$$

$$x^2 + 2x - 4x - 8 = 0$$

$$\vdots$$

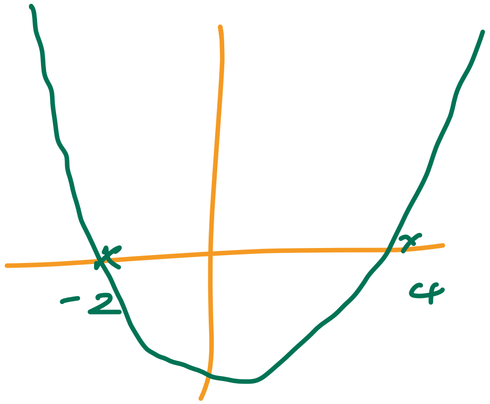
$$(x+2)(x-4) = 0$$

NFL

$x + 2 = 0$ or $x - 4 = 0$

$\underline{\underline{x = -2}}$ $\quad$ $\underline{\underline{x = 4}}$

-8	
1	-8
2	-4



Examples: Solving equations with quadratic trinomials

Solve $x^2 - 8x + 15 = 0$

$$x^2 - 8x + 15 = 0$$

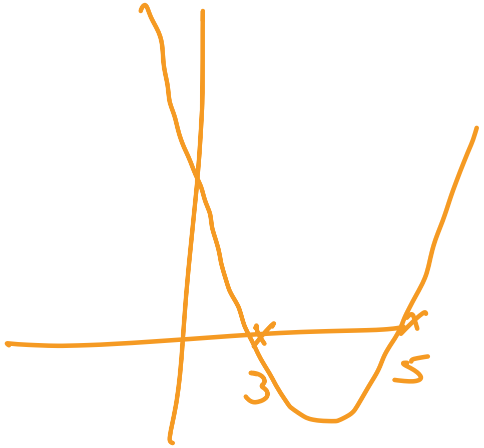
$$\vdots$$

$$(x-3)(x-5) = 0$$

$$\therefore \quad x - 3 = 0 \quad \text{or} \quad x - 5 = 0$$

$$x = 3 \qquad\qquad x = 5$$

$+15$	
-1	-15
-3	-5

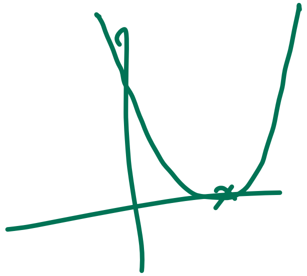


Example: Solving perfect squares

Just because they are a perfect square, doesn't mean it gets treated any differently!

Like Lord of the Rings, lets have one rule to bind them all ;)

Solve $x^2 - 8x + 16 = 0$



$$x^2 - 8x + 16 = 0$$

$$\vdots$$

$$(x-4)(x-4) = 0$$

$$(x-4)^2 = 0$$

$$\therefore\ x - 4 = 0 \quad \text{or} \quad x - 4 = 0$$

$$\underline{\underline{x = 4}} \qquad \underline{\underline{x = 4}}$$

$+16$

~~-1~~	~~-16~~
~~-2~~	~~-8~~
$\boxed{-4}$	$\boxed{-4}$

Example: Solving perfect squares

Just because they are a perfect square, doesn't mean it gets treated any differently!

Like Lord of the Rings, lets have one rule to bind them all ;)

Solve $x^2 = x + 6$

$$x^2 = \cancel{x} + \cancel{6}$$

$$x^2 - x - 6 = 0$$

$$(x+2)(x-3) = 0$$

$$\therefore x + 2 = 0 \quad \text{or} \quad x - 3 = 0$$

$$x = -2 \qquad x = 3$$

-6

~~1~~	~~-6~~
2	-3

Trick questions to test if you have understood

Remember, the whole point of doing Maths is to see if you have understood it.

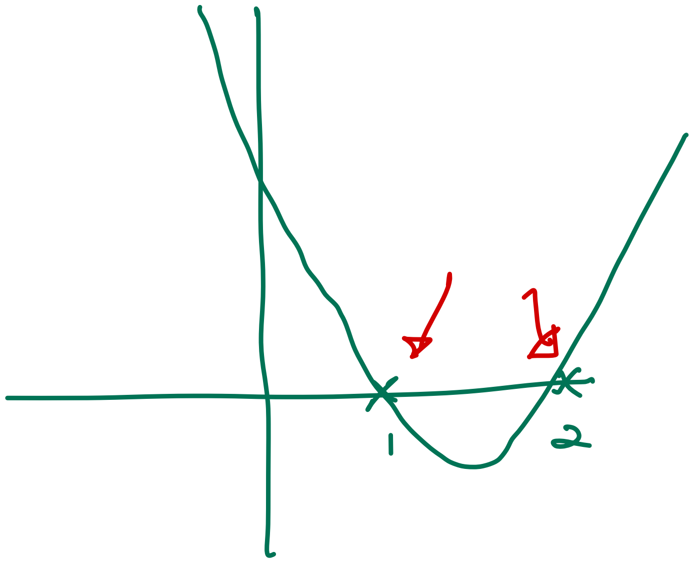
So, I am now going to give you two solutions and ask, can you back track and tell the equation it came from?

Solutions are: $x = 1$ and $x = 2$

$$(x-1)(x-2)$$

$$= x^2 - 2x - x + 2$$

$$= x^2 - 3x + 2$$



Trick questions to test if you have understood

Are there two ways to complete the following question?

Solve $2x^2 - 2x - 12 = 0$

$$2x^2 - 2x - 12 = 0$$

$$x^2 - x - 6 = 0 \quad \ldots\ldots \checkmark$$

Questions to complete:

The questions I would like you to complete for this lesson are:

Exercise 10C Solving $x^2 + bx + c = 0$
Questions: 2agmp. 3beh, 4fhj, 51d, 6bde, 7abe, 10

Extension: 11, 12adg

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