Solving exponentials equations using logarithms

Year 10 Mathematics

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Learning Objectives

By the end of the lesson, I would hope that you have an understanding and be able to apply to questions the following concepts:

- To know how to solve exponential equations by rewriting in logarithmic form using the given base
- To be able to solve an exponential equation using base 10
- To be able to use technology to evaluate logarithms

Recap of past learning

If you are watching this in order, you will now know:

- How to convert “normal numbers” into logarithmic numbers
- What a log graph looks like
- How to manipulate logs by:
    - Adding
    - Multiplying
    - Raising them to a power
- As well as some of the more important log rules

All of the above means we can now start to use the learning to **solve equations**.

$$\log_a x + \log_a y = \log_a(xy)$$

$$\log_a x - \log_a y = \log_a\left(\frac{x}{y}\right)$$

$$\log_a(x^n) = nlog_a(x)$$

$$\log_a 1 = 0$$

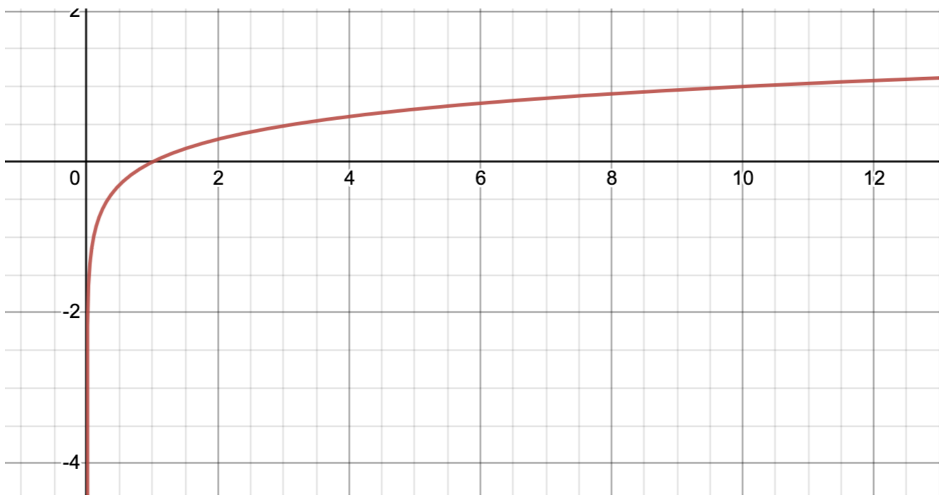
$$\log_a a = 1$$

$$\log_a\left(\frac{1}{x}\right) = \log_a(x^{-1}) = -\log_a x$$

$$a^x = y$$

$$\updownarrow$$

$$\log_a(y) = x$$

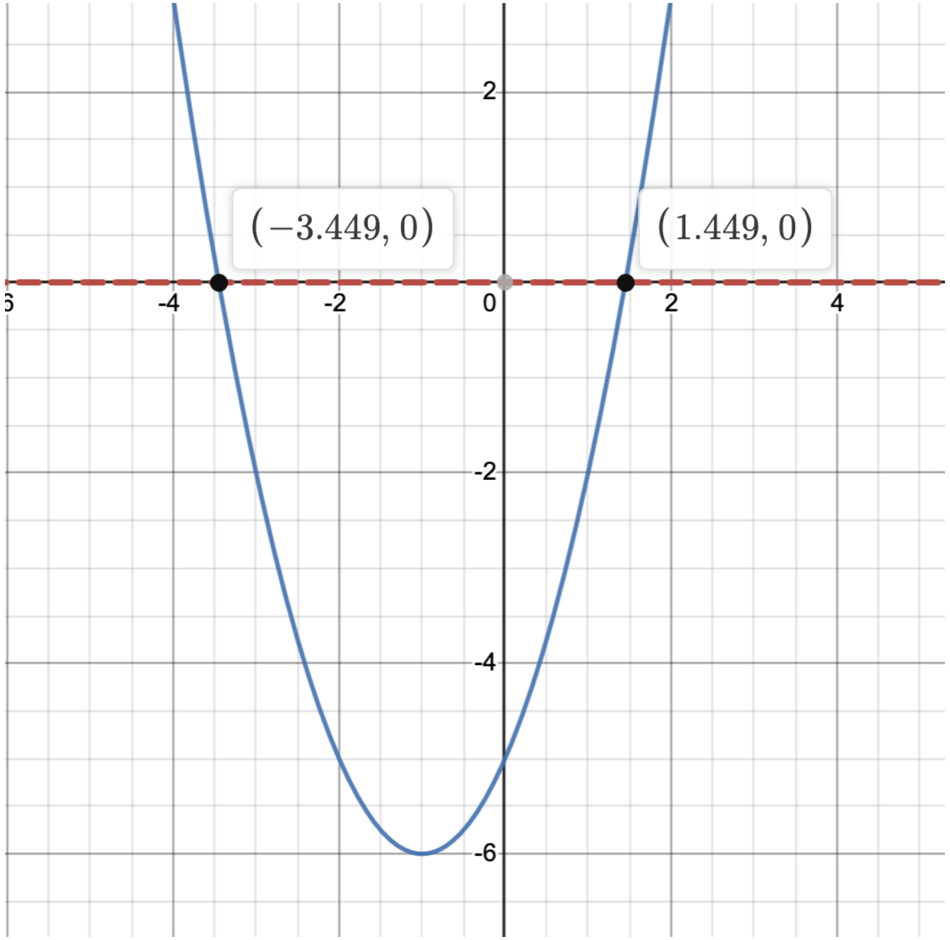


What is an equation?

This is where we have an expression equals to something. Generally, we want to be able to find the value of a pronumeral (or unknown) which will make the equation true on both sides.

It's useful to think of this as finding the **points of intersection** of two graphs!

When we have something like $x^2 + 2x - 5 = 0$ what we are really doing is finding the crossing points of the graph $x^2 + 2x - 5$ and the line $y = 0$



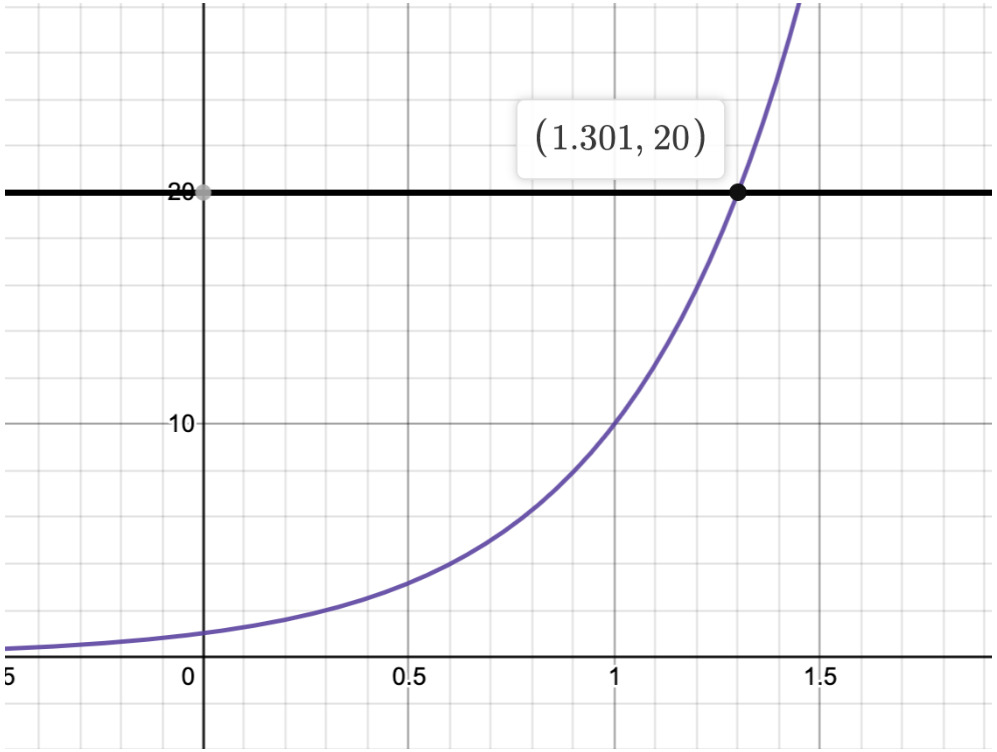
Finding the solutions to harder questions

It’s not easy to find the solutions using pencil and paper methods to the following:

$$10^x = 20$$

We could draw a graph!

$y = 10^x \quad y = 20$



Hold on .. This look nothing like the logarithm graph!!!

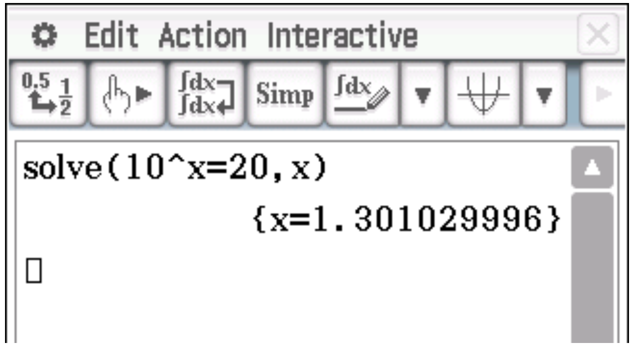
What if we don't have access to Desmos?

Then we can use the CAS to solve this for us:

$10^x = 20$

$y = 10^x$

$y = 10^{1.30103}$



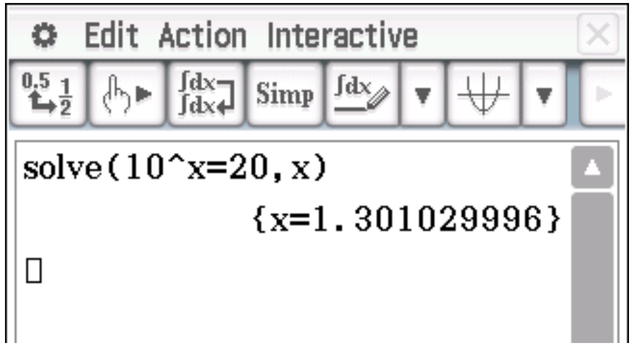
Another way to solve this

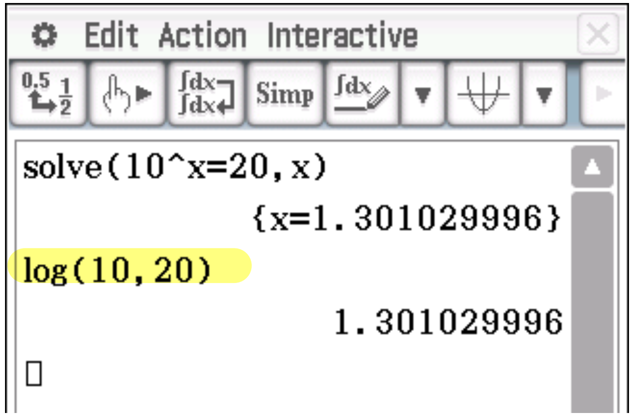
Then we can use the CAS to solve this for us:

$10^x = 20$
$x = \log_{10} 20$

$10^x = 20$

$\log_{10} 20 = x$





Example: Solving using the given base

Solve the following using the given base. Round your answer to three decimal places.

a $2^x = 7$ **b** $50 \times 1.1^x = 100$

a. $2^x = 7$

$\log_2 7 = x$

$x = 2.807$

b. $50 \times 1.1^x = 100$

$1.1^x = 2$

$\log_{1.1} 2 = x$

$x = 7.272$

Example: Solving using base 10

Solve using base 10 and evaluate, correct to three decimal places.

a $3^x = 5$

b $1000 \times 0.93^x = 100$

a.

$$\log_{10} 3^x = \log_{10} 5$$

$$x \cdot \log_{10} 3 = \log_{10} 5$$

$$x = \frac{\log_{10} 5}{\log_{10} 3}$$

$$x = 1.465$$

b.

$$1000 \times 0.93^x = 100$$

$$0.93^x = 0.1$$

$$\log_{10} 0.93^x = \log_{10} 0.1$$

$$x \cdot \log_{10} 0.93 = \log_{10} 0.1$$

$$x = \frac{\log_{10} 0.1}{\log_{10} 0.93}$$

$$= 31.729$$

Change of base formula

This is a little outside of the “standard” Year 10 course, but it’s interesting to look at something called the “Change of Base” formula

$$a^x = y$$

$$\log_b a^x = \log_b y$$

$$x \cdot \log_b a = \log_b y$$

$$x = \frac{\log_b y}{\log_b a}$$

$$\log_a y = \frac{\log_b y}{\log_b a}$$

$$a^x = y$$

$$\log_a y = x$$

Example: Change of base formula

Let's see how this might be applied:

Use the change of base formula to write the following with a base of 10:

$\log_2 7$

$$\log_2 7 = \frac{\log_{10} 7}{\log_{10} 2}$$

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