Selections

Year 11

Mathematical Methods

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Learning Objectives

By the end of the lesson, I hope that you understand and can apply the following to a range of questions from the Year 11 Mathematical Methods course.

- Understand what it means to be a selection
- Understand how to find the number of selections which might be made.
- Understand how to find selections of any size.
- Remember and use Pascals Triangle
- Understand what Pascals Identity is.

RECAP

In the last lesson we looked at how we can select and arrange a number of items. What happens if we are only interested in the number of different groups of objects which can be selected.

In this case the order would be unimportant.

$3.2.1$

A B C

1. ABC
2. CBA

⋮

$\underline{\underline{6}}$

I'm going to learn my ABC's again

Actually, in this case, I'm only going to learn my A, B and C. I get confused after this!

Consider that I want to see how many groups of two I can make with the letters A,B and C.

AB BA AC CA BC CB

$\underline{3}\ \underline{2} = \underline{\underline{6}}$

I’m going to learn my ABC’s again

What would change if the order wasn’t important.

AB, BA, AC, CA, BC, CB

In this case, there are only three distinct selections when the order doesn’t matter.

Example 1

Four flavours of ice-cream – vanilla, chocolate, strawberry and caramel – are available at the school canteen. How many different double-scoop selections are possible if two different flavours must be used?

Van Choc

Van Straw

Van Caramel

Chol Straw

Choc Car

Straw Car

6

Let’s start using the word combinations

When we make choices without regard to order, we call them selections or **combinations**.

We use the notation shown below where *n* is the number of objects (where order isn’t important) and *r* is the size of each group.

$^{n}P_{r}$

$$^{n}C_{r}$$

Finding a formula

There must be a formula, right?

$$nCr = \frac{nPr}{r!} = \frac{n!}{r!\,(n-r)!}$$

If we look back at the example relating to the colour on the Olympic flag, we had 8 colours and only 5 rings.

How many choices of colours are there? How many way are there of arranging them?

$8!$

$\underline{8}\ \underline{7}\ \underline{6}\ \underline{5}\ \underline{4}$

R O Y G B

B Y O R G

$\frac{8!}{3!\,5!}$

5 4 3 2 1 $5!$

Example 2

Consider the situation from the first Example again: If four flavours of ice-cream are available, how many double-scoop selections are possible if two different flavours must be used?

$n = 4$

$r = 2$

$$^{n}C_{r} = \frac{4!}{2!\,2!} = \frac{\overset{2}{\cancel{4}} \times 3 \times \cancel{2 \times 1}}{\cancel{2 \times 1} \times \cancel{2 \times 1}} = \underline{\underline{6}}$$

Example 3

A team of three boys and three girls is to be chosen from a group of eight boys and five girls. How many different teams are possible?

$$\begin{array}{ccc} B & & G \\ {}^{8}C_{3} & \times & {}^{5}C_{3} \end{array}$$

$$\frac{8!}{5!\,3!} \times \frac{5!}{2!\,3!}$$

$$56 \times 10$$

$$= \underline{\underline{560}}$$

Selections of any size

What would happen if we needed to count all the possible combinations of any size from a group of n objects?

We are told the following:

For n objects:

$${}^nC_0 + {}^nC_1 + {}^nC_2 + \cdots + {}^nC_{n-1} + {}^nC_n = 2^n$$

A B C

$8 = {}^3C_0 + {}^3C_1 + {}^3C_2 + {}^3C_3$

${}^3C_0$	${}^3C_1$	${}^3C_2$	${}^3C_3$
1	3	3	1
O	A	AB	ABC
	B	AC	
	C	BC	

$\therefore {}^3C_0 + {}^3C_1 + {}^3C_2 + {}^3C_3 = 2^3$

${}^nC_0 \ldots + {}^nC_n = 2^n$

$2 \times 2 \times 2 = 8 = 2^3$

Example 4

Nick is making an invitation list for his party, and has seven friends to choose from. If he may choose to invite any number of friends (from one to all seven), how many possible party lists does he have? (Assume he will invite at least one person to his party.)

$$^7C_0 + {}^7C_1 + {}^7C_2 + {}^7C_3 + {}^7C_4 + {}^7C_5 + {}^7C_6 + {}^7C_7 = 2^7$$

$$^7C_1 + \ldots {}^7C_7 = 2^7 - {}^7C_0$$

$$= 2^7 - 1$$

$$= 128 - 1$$

$$= \underline{\underline{127}}$$

Pascal's triangle and identity

We remember, from a previous topic area, that Pascal's triangle can be really helpful when finding the coefficients for expansion of brackets.

$n = 0$:						$^0C_0$						
$n = 1$:					$^1C_0$		$^1C_1$					
$n = 2$:				$^2C_0$		$^2C_1$		$^2C_2$				
$n = 3$:			$^3C_0$		$^3C_1$		$^3C_2$		$^3C_3$			
$n = 4$:		$^4C_0$		$^4C_1$		$^4C_2$		$^4C_3$		$^4C_4$		
$n = 5$:	$^5C_0$		$^5C_1$		$^5C_2$		$^5C_3$		$^5C_4$		$^5C_5$	

						1						
					1		1					
				1		2		1				
			1		3		3		1			
		1		4		6		4		1		
	1		5		10		10		5		1	
1		6		15		20		15		6		1

$n = 5$

$r = 2$

$$^nC_r = {}^{n-1}C_{r-1} + {}^{n-1}C_r \quad \text{for } 0 < r < n$$

$$^5C_2 = {}^4C_1 + {}^4C_2$$

This is Pascal's Identity

Learning Objectives: Revisited

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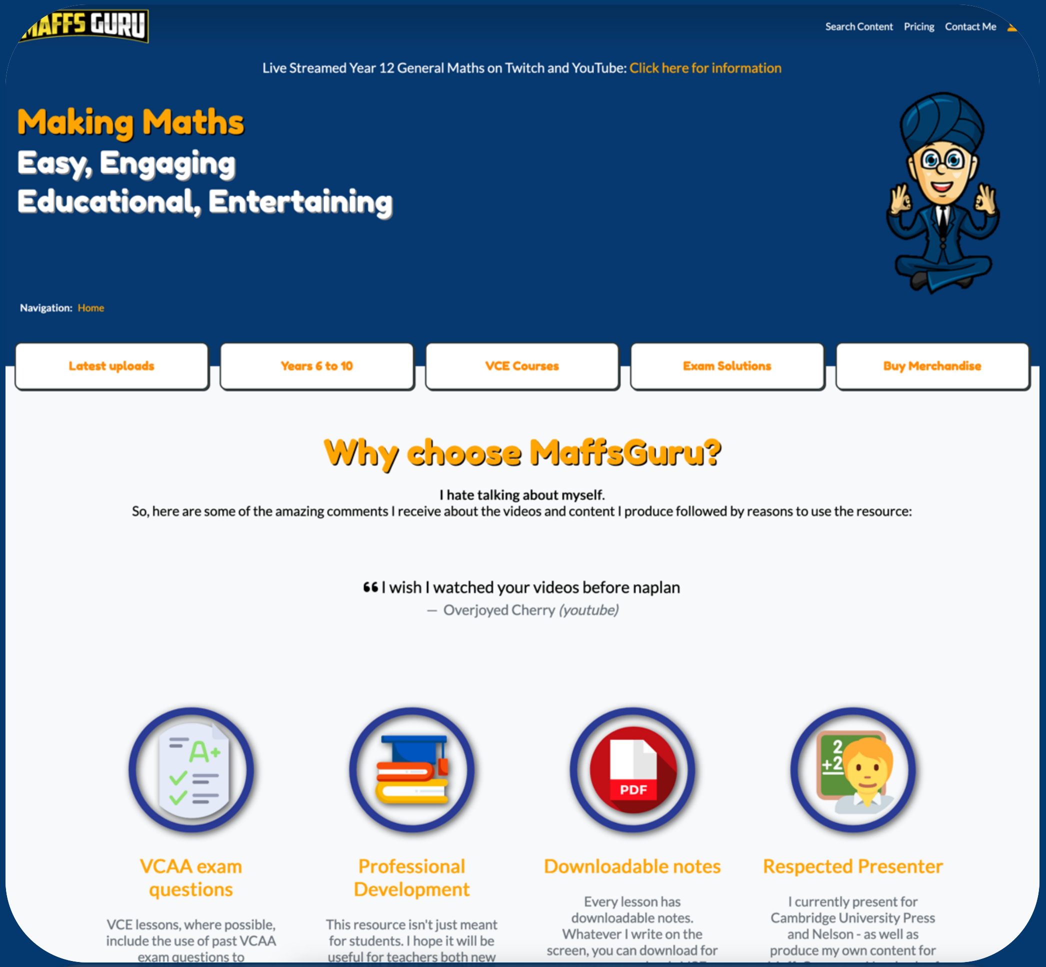
Questions to complete

The following are the minimum number of questions you are expected to answer. There is nothing wrong with answering more!

Ex 10C
Questions: TBA



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