Laws of logarithms

Year 10 Mathematics

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Learning Objectives

By the end of the lesson, I would hope that you have an understanding and be able to apply to questions the following concepts:

- To know how to combine logarithms with the same base using the logarithm laws for addition and subtraction
- To know properties of logarithms involving powers and the logarithm of 1
- To be able to use logarithm properties to simplify expressions

Recap of past learning

In the last lesson we looked at what a logarithm was and how we can express larger numbers as smaller numbers. This normally helps us to graphs them. The “original numbers” don’t change, they just become more manageable.

The important algebra is shown on the right.

We are not going to see how we can use algebra to add, subtract, multiply and divide logarithms in much the same way as we did with the index laws.

$$a^x = y$$

$$\log_a(y) = x$$

$10^4 = 10000$

$\log_{10}(10\,000) = 4$

Logarithm Law 1

We love a law in Mathematics! What I show below, you only need to know the final result. But it's interesting to see where it came from!

Index Law 1: $a^m \times a^n = a^{m+n}$

$x = a^m \qquad y = a^n$

$\log_a x = m \qquad \log_a y = n$

$a^{m+n} = xy$

$\log_a (xy) = m + n$

$m + n = \log_a x + \log_a y$

$\log_a x + \log_a y = \log_a (xy)$

Logarithm Law 2

That was cool … said no one ever.

Let's just go straight to the rule then:

$$\log_a(x) - \log_a(y) = \log_a\left(\frac{x}{y}\right)$$

Logarithm Law 3

Oh yes … there is another one!

$$\log_a(x^n) = nlog_a(x)$$

This rule is used over and over and over again!

Other important properties

There are other important things we need to know about logarithms and they are listed below and explained on the next slide.

$$\log_a 1 = 0$$
$$\log_a a = 1$$
$$\log_a \left(\frac{1}{x}\right) = \log_a(x^{-1}) = -\log_a x$$

$a^0 = 1$

$a^1 = a$

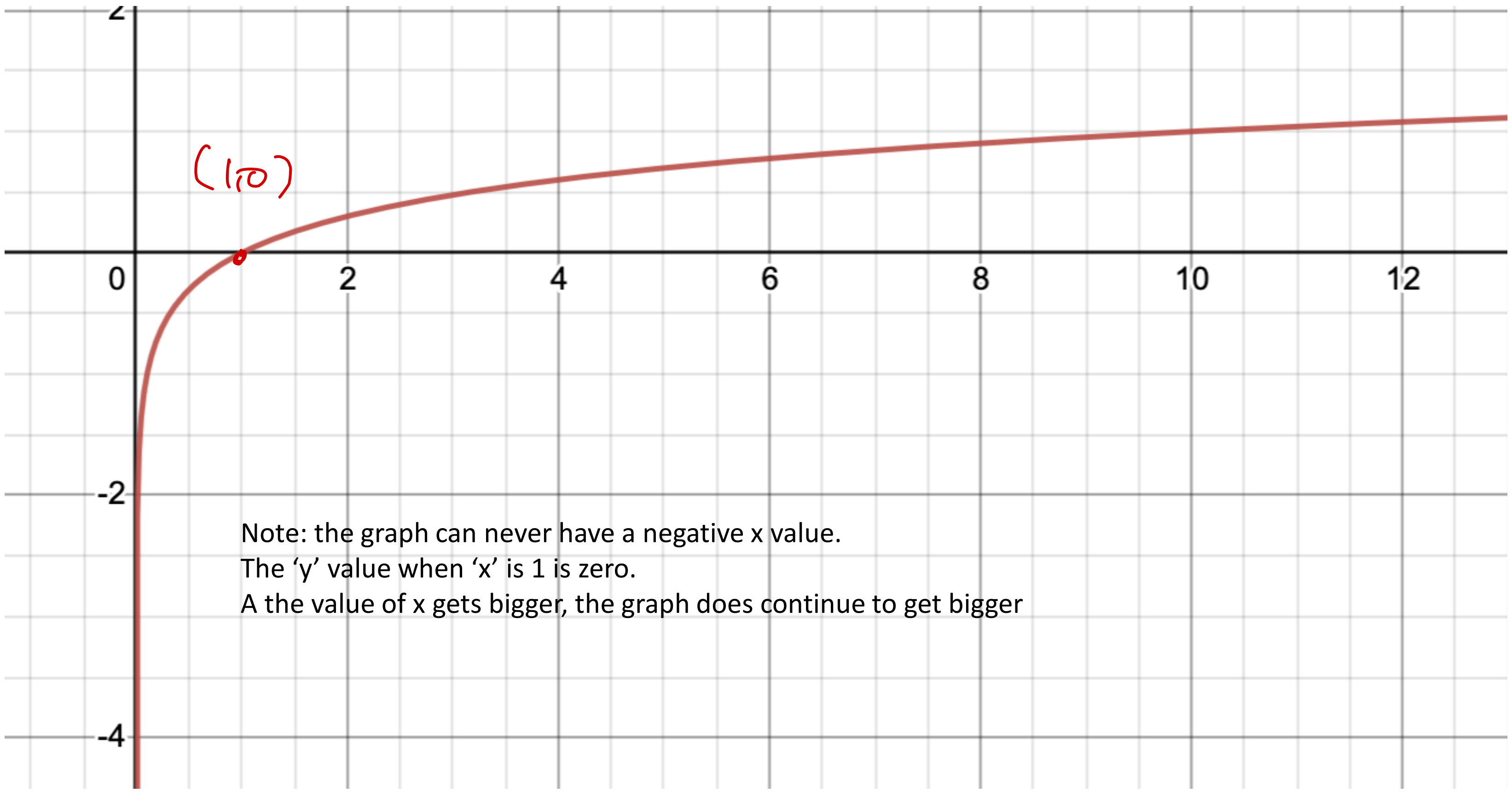
$\frac{1}{x} = x^{-1}$

$\log_a(x^{-1}) = -1.\log_a(x)$

$= -\log_a(x)$

What does a logarithmic graphs look like?

This becomes really important in later parts of this course (and later). So, why not have a look at one now!



Note: the graph can never have a negative x value.
The ‘y’ value when ‘x’ is 1 is zero.
A the value of x gets bigger, the graph does continue to get bigger

Example: Simplifying logarithmic expressions

Simplify the following.

a $\log_a 4 + \log_a 5$ **b** $\log_a 22 - \log_a 11$ **c** $3\log_a 2$

a. $\log_a 4 + \log_a 5 = \log_a(4 \times 5)$

$= \log_a(20)$

b. $\log_a 22 - \log_a 11 = \log_a\left(\frac{22}{11}\right) = \log_a 2$

c. $3\log_a 2 = \log_a 2^3 = \log_a 8$

Example: Evaluating logarithms

Simplify and evaluate the following expressions.

a $\log_2 1$ **b** $\log_5 5$ **c** $\log_6 \frac{1}{36}$ **d** $\log_2 6 - \log_2 3$

a. $\log_2 1 = x$

$2^x = 1$

$x = 0$

b. $\log_5 5 = x$

$5^x = 5$

$x = 1$

d. $\log_2 \left(\frac{6}{3}\right)$

$= \log_2 2$

$= 1$

b. $\log_6 \left(\frac{1}{36}\right) = \log_6 \left(36^{-1}\right)$

$= -\log_6 36$

$= -2$

Example: More complex logarithm questions

There are so many questions which might be given but what would you do with the following

$$\log_2 \sqrt{8}$$

$$= \log_2 8^{1/2}$$

$$= \frac{1}{2} \cdot \log_2 8$$

$$= \frac{1}{2} \cdot 3$$

$$= \frac{3}{2}$$

$$\sqrt[3]{x} = x^{1/3}$$

$$\sqrt[4]{x} = x^{1/4}$$

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Examples have been extracted, with permission, from the Cambridge Essentials (Year 10) Textbook

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