Arrangements

Year 11

Mathematical Methods

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Learning Objectives

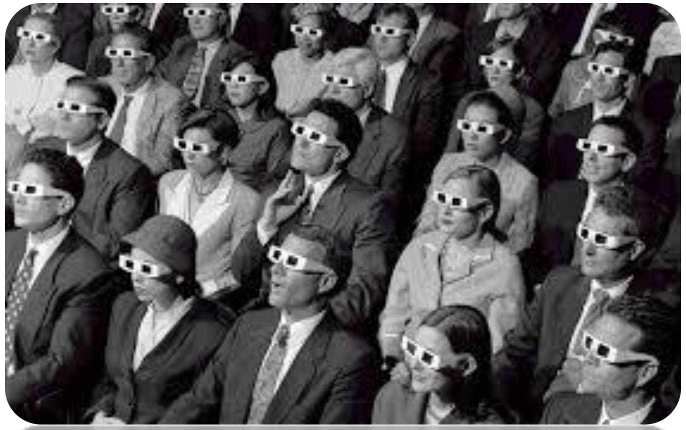
By the end of the lesson, I hope that you understand and can apply the following to a range of questions from the Year 11 Mathematical Methods course.

- Understand what it means to be an arrangement
- Understand how to find the number of arrangements
- Finding the number of arrangements with and without restrictions

RECAP

This is another new section of the Mathematical Methods Units 1 and 2 course. It looks at the number of ways we might be able to arrange a distinct number of objects.

For example, imagine you were heading to the movies with 3 friends. How many ways are there for you all to sit down if there are only 3 seats free in the movie theatre.



What is an arrangement?

This is the number of different orders a distinct number of objects can be placed. If we think back to the example a moment ago, we can see there might be something of a pattern we can use to help us.

Note: The wording of the question is really important!

Note: Arrangements are also called permutations.

$\boxed{3}$

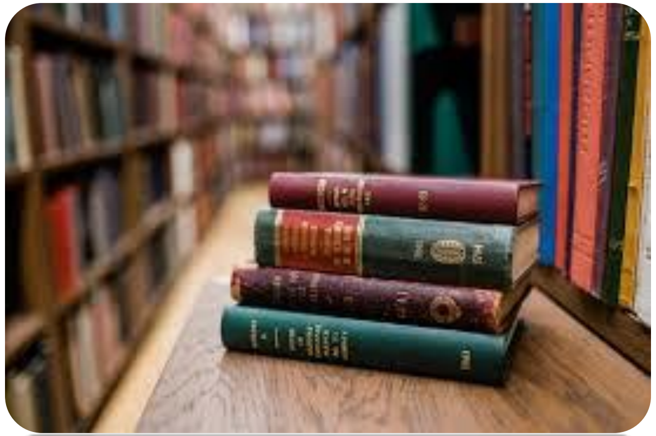
A, B, C

$\underline{3} \times \underline{2} \times \underline{1} = \underline{\underline{6}}$

A B C
A C B
B A C
B C A
C A B
C B A

Example 1

How many ways are there of arranging four different books on a shelf?



$\underline{\underline{4}}$

$\underline{4} \times \underline{3} \times \underline{2} \times \underline{1}$

$= \underline{\underline{24}}$

A rule

In general, if n objects are arranged in a row, then there are n choices for the first position, $n - 1$ choices for the second position, $n - 2$ choices for the third position, and so on, until 1 is reached.

Thus, the number of ways of arranging n objects in a row is

$$n \times (n-1) \times (n-2) \times (n-3) \times \cdots \times 2 \times 1$$

4

n

$\underline{n}$ $\underline{n-1}$ $\underline{n-2}$ $\underline{n-3}$ . . . . . $\underline{1}$

Example 2

A photo is to be taken of a group of 12 students. How many ways are there of arranging the group if they all sit in a row?

$\underline{12}\ \underline{11}\ \underline{10}\ \underline{9}\ \underline{8}\ \underline{7}\ \ldots\ 1$

$= 479\,001\,600$

Factorial notation!

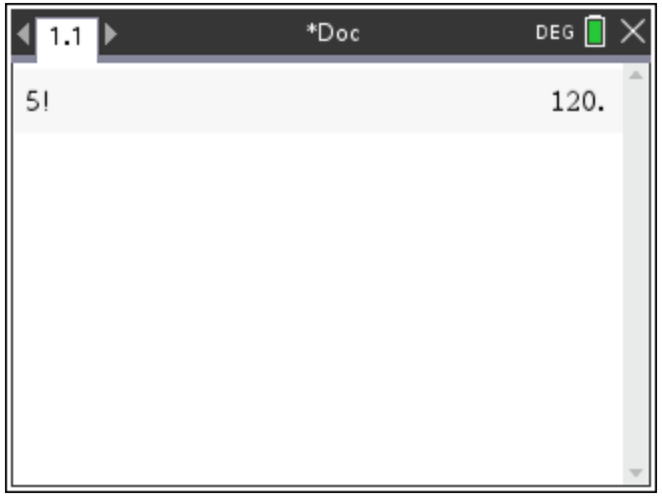
There are jokes which circulate on the internet which relate to the use of factorials.

Basically, we use an ! mark to stand for the product of whole numbers leading down to 1.

So, for example, $5! = 5 \times 4 \times 3 \times 2 \times 1$

We can use the CAS to help us find factorials!

$5! = 5 \times 4 \times 3 \times 2 \times 1$



Example 3

A painter is to paint the five circles of the Olympic flag. He cannot remember the colours to use for any of the circles, but he knows they should all be different. He has eight colours of paint available. In how many ways can he paint the circles on the flag?

Note, this is a trick! There are 8 choices, but only 5 circles.

8

$\underline{8}\ \underline{7}\ \underline{6}\ \underline{5}\ \underline{4} = 6720$

A formula

We love a good formula in Mathematics and so we are going to look at where the following one comes from:

$$number\ of\ arrangements\ of\ n\ objects\ in\ groups\ of\ size\ r, nPr = \frac{n!}{(n-r)!}$$

8 cols

5 rings

$${}^{n}P_{r} = \frac{n!}{(n-r)!} = \frac{8!}{3!} = \frac{8\times7\times6\times5\times4\times\cancel{3\times2\times1}}{\cancel{3\times2\times1}}$$

Example 4

Find the number of different four-digit numbers that can be formed from the digits 1, 2, 3, 4, 5, 6, 7, 8, 9, if each digit:

a can only be used once **b** can be used more than once.

$n = 9$

$r = 4$

a. $^{n}P_{r} = \dfrac{9!}{(9-4)!} = \dfrac{9!}{5!} = 9 \times 8 \times 7 \times 6 = 3024$

b. $\underline{9}\ \underline{9}\ \underline{9}\ \underline{9} = 9^4 = 6561$

What does it mean to be 0!

It might be wise to accept this as being 1.

$n = r$

$n \Rightarrow n!$

$${}^{n}P_{r} = \frac{n!}{(n-r)!}$$

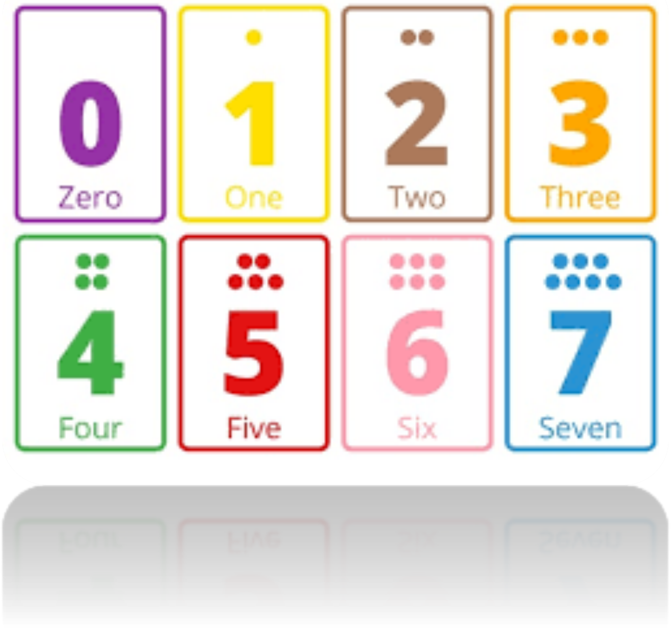
$${}^{n}P_{n} = \frac{n!}{(n-n)!} = \frac{n!}{0!} = \frac{n!}{1}$$

Arrangements with restrictions

How many different even four-digit numbers can be formed from the digits 1, 2, 3, 4, 5, 6, 7, 8, if each digit can only be used once?

The best way to do this is to use our tried and trusted boxes.

Note: Even numbers must end in ~~0,~~ 2, 4, 6 or 8.



$$7 \quad 6 \quad 5 \quad \boxed{4}$$

$$= 840$$

Learning Objectives: Revisited

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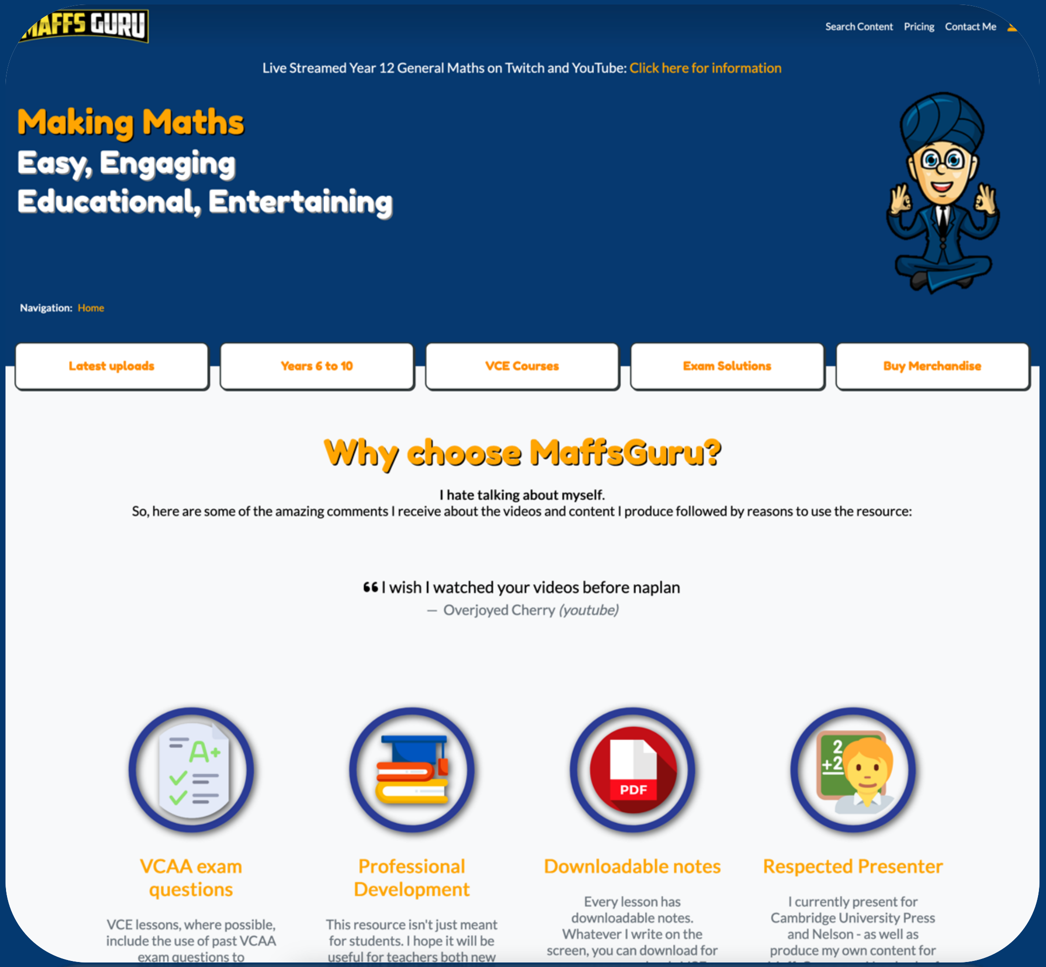
Questions to complete

The following are the minimum number of questions you are expected to answer. There is nothing wrong with answering more!

Ex 10B
Questions: TBA



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