Introducing logarithms

Year 10 Mathematics

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Learning Objectives

By the end of the lesson, I would hope that you have an understanding and be able to apply to questions the following concepts:

- To understand the form of a logarithm and its relationship with index form
- To be able to convert between equivalent index and logarithmic forms
- To be able to evaluate simple logarithms both with and without technology
- To be able to solve simple logarithmic equations

Recap of past learning

This is such an interesting subject as it leads very much into the work we will be using in Year 11 and 12 (if you're going to do Mathematical Methods here in Australia). Understanding not only what a logarithm is, but also polynomials will make life so much easier.

As this is the start of a new module of work, there isn't really a recap of past learning as everything you have learned before this point is going to be really helpful.

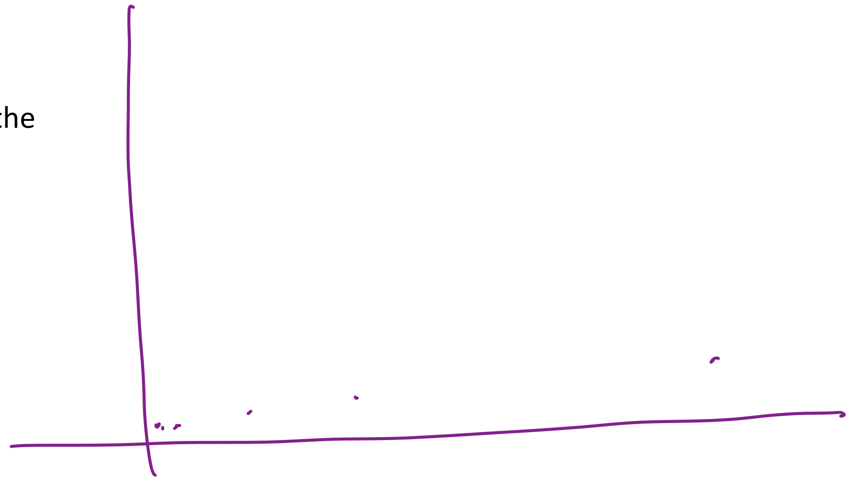
Note: This content is algebra heavy and you will need to have a good understanding of how to use it to succeed.

What are logarithms?

They are basically a way for us to be able to express really big numbers in a smaller way! So important are they, that they are also in the Year 12 General Maths course! So, I'm going to explain it the same way as it do for that course.

Imagine I had to plot the following data items on a single graph. Imagine they might need to be placed on a dot plot.

1, 10, 100, 1000, 10 000, 100 000, 1 000 000



Knowing that a graph to have an even scale, we need to try and fit all the numbers between 1 and 1 000 000 on the scale.

Let's have a go using just pencil and paper methods …

Let's redefine the number system

We can think of the numbers in a different way because we know we can write each of them as a power of 10.

1, 10, 100, 1000, 10 000, 100 000, 1 000 000

10^0 10^1 10^2 10^3 10^4 10^5 10^6 . . .

0 1 2 3 4 5 6

$\log_{10}(10) = 1$

If we then define a number system where we use the powers of 10 instead of the numbers, then we have a nice way to scale them.

This also works for smaller numbers too

We can think of the numbers in a different way because we know we can write each of them as a power of 10.

0.01, 0.1, 1, 10, 100, 1000, 10 000, 100 000, 1 000 000

10^{-2} 10^{-1} 10^{0} 10^{1} 10^{2}

-2 -1 0 1 2 3

$\log_{10}(0.01) = -2$

$10^{-2} = 0.01$

Changing a ”raw number” into a logarithm

It’s a way of writing any number as a power of 10.

We write it using $\log_{10}(x)$ notation.

For example:

$$\log_{10}(1000) = 3$$

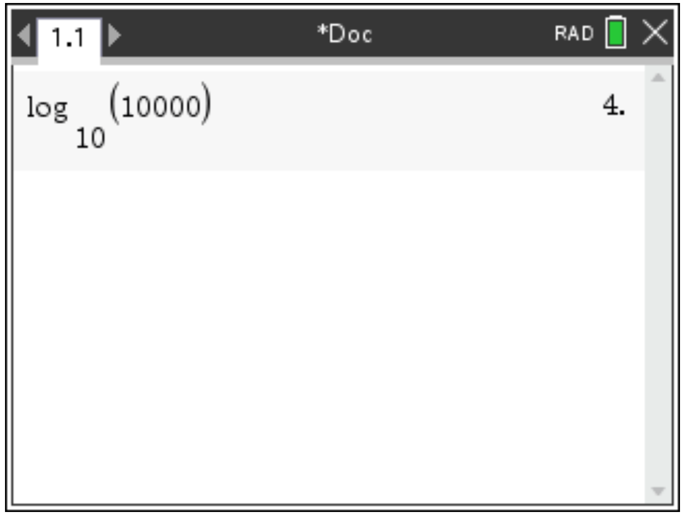
I read this as 10 to the power of what is 1000?

Using the CAS: Normal to log base 10

Your CAS can work out log numbers for you.

In the example shown we are converting from a “normal” number into a ”log base 10” number.

This is important as, what we can do forwards we can do backwards too!



Log base 10 to normal

We know that the final number of the log conversion is really a power of 10.

So, we can use this to help us go from log numbers to normal numbers!

Raising 10 to this power will give you the normal number

$$\log_{10}(1000) = 3$$

$$10^5 = x$$

$$\log_{10}(x) = 5$$

$$10^5 = 100000$$

So, we are now able to convert from log base 10 to normal and back again.

Using algebra

We need to become more comfortable with using algebra and looking at things in a more algebraic form.

$$a^x = y$$

$$\updownarrow$$

$$\log_a(y) = x$$

Example: Writing equivalent statements using logarithms

Write an equivalent statement to the following.

a $\log_{10} 1000 = 3$

b $2^5 = 32$

a. $\log_{10} 1000 = 3$

$10^3 = 1000$

$2^5 = 32$

$\log_2 32 = 5$

Example: Evaluating logarithms

a Evaluate the following logarithms.

i $\log_2 8$ **ii** $\log_5 625$

b Evaluate the following.

i $\log_3 \frac{1}{9}$ **ii** $\log_{10} 0.001$

c Evaluate, correct to three decimal places, using a calculator.

i $\log_{10} 7$ **ii** $\log_{10} 0.5$

0.845 -0.301

$\log_2 8 = x$

$2^x = 8$

$x = 3$

$\log_5 625 = x$

$5^x = 625$

$x = 4$

$\log_3 \frac{1}{9} = x$

$3^x = \frac{1}{9}$

$3^x = 9^{-1}$

$3^x = (3^2)^{-1}$

$3^x = 3^{-2}$

$x = -2$

$\log_{10} 0.001 = x$

$10^x = 0.001$

$10^x = \frac{1}{1000}$

$10^x = 1000^{-1}$

$10^x = (10^3)^{-1}$

$10^x = 10^{-3}$

$x = -3$

Example: Solving simple logarithmic equations

Find the value of x in these equations.

a $\log_4 64 = x$

b $\log_2 x = 6$

$\log_4 64 = x$

$4^x = 64$

$x = 3$

$\log_2 x = 6$

$2^6 = x$

$x = 64$

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