Unions and Intersections

Tuesday, 30 October 2018 6:44 PM

By the end of the lesson I would hope that you have an understanding of the following. I would also hope that you can apply the understanding to a number of different questions and question types

- Understand what a Venn Diagram is
- Know how to draw a Venn Diagram to represent the outcomes of probability experiments
- What it means (in the context of a Venn Diagram) for a Union
- What it means (in the context of a Venn Diagram) for an Intersection
- How to use the concepts of Unions and Intersections to calculate probabilities

RECAP:

We have been looking at the concepts behind probability.
In essence, the work is really easy.
We are now going to ramp up the complexity just a little bit.

Firstly, recap some language:

- **Sample Space**: The complete list of outcomes from an experiment
- **Trial**: The act of performing a single experiment
- **Outcome**: Result from an experiment
- **Event**: A collection of outcomes e.g. Pr(even number)
- **Equally likely**: The outcomes have the same chance of occurring

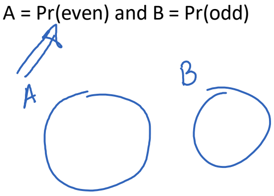
$A = \{1, 2, 3, 4, 5, 6\}$

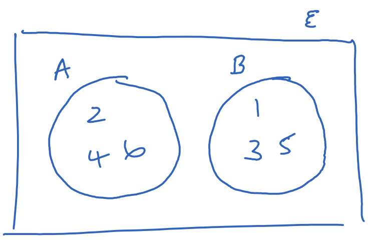
The Venn Diagram

Venn Diagrams are used to pictorially represent events.
They are pretty simple really.
Here are some examples:

Two events: Rolling a Die
A = Pr(even) and B = Pr(odd)

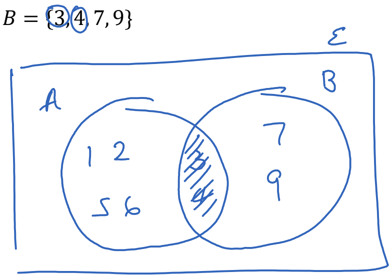
$\{1,2,3,4,5,6\}$

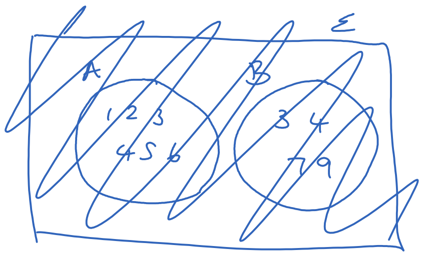




Mutually Exclusive

Two Events:
$A = \{1, 2, 3, 4, 5, 6\}$ *and* $B = \{3, 4, 7, 9\}$

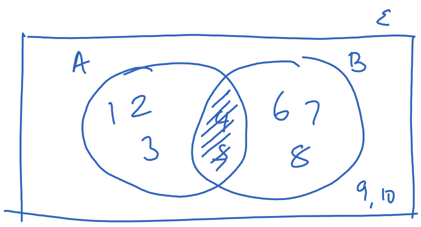




Notice with the second event there is an overlap where the two events share some numbers.
This is called the **Intersection** of the data.
In America roads **meet** at **intersections**.

Other examples of Intersections

In the following example, notice the numbers both in the circles and outside the circles.



$A = \{1, 2, 3, 4, 5\}$

$B = \{4, 5, 6, 7, 8\}$

$\varepsilon = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

The way we express INTERSECTION in Mathematics is:

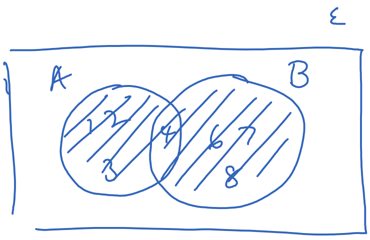
$A \cap B$

State of the Union

Lots of references to America at the moment, but they have the State of the Union address.
When we unite people ... they all come together.
Union means all.

When we have the Union of the data we look at all the data joined together.

This can be shown with the following Venn Diagram:



$A = \{1,2,3,4\}$

$B = \{4,6,7,8\}$

$A \cup B = \{1,2,3,4,6,7,8\}$

The way we express UNION in Mathematics is:

$A \cup B$

Maths is a BIG FAT TRICK and will try and use alternative notation

Venn Diagrams are awesome ...
We can describe the areas on a Venn Diagram in different ways.
Firstly though ... we need to know what a complement is ...

compliment

noun
/ˈkɒmplɪm(ə)nt/

1. a polite expression of praise or admiration.
"she paid me an enormous compliment"
synonyms: flattering remark, tribute, accolade, commendation, bouquet, pat on the back, encomium; More

verb
/ˈkɒmplɪmɛnt/

1. politely congratulate or praise (someone) for something.
"he **complimented** Erika **on** her appearance"

Sadly, this is not what I meant!

I meant complement ... **This simply means the opposite of something.**

Hence, if $Pr(A) = 0.4$ then it's complement is $Pr(\bar{A}) = 0.6$

Notice the little mark above it.
This means it's the complement.

We can also describe it as **not the probability of getting A**.

Complement

$Pr(A') = 1 - Pr(A)$

$Pr(A) = 0.4$

$Pr(\bar{A}) = 1 - Pr(A)$

This means it's the complement.

We can also describe it as **not the probability of getting A**.

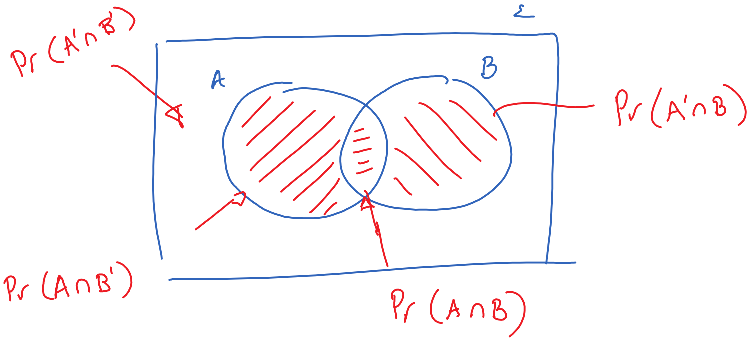
Knowing this we can now look at our Venn Diagram again ....

$Pr(A) = 0.4$ $\quad Pr(\bar{A}) = 1 - Pr(A)$

$Pr(\bar{A}) = 0.6$

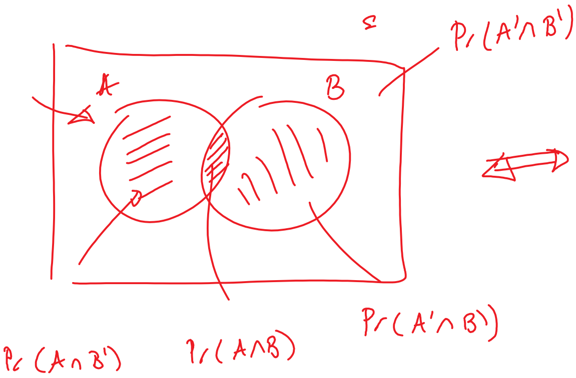
$Pr(\text{rain}) = 0.2$

$Pr(\overline{\text{rain}}) = 0.8$



Using Tables to describe Venn Diagrams and Probabilities/Numbers of outcomes

There are these really funky things called probability tables (or Karnaugh Maps) which are just da bomb!



n

∩	A	A'	
B	A∩B	A'∩B	
B'	A∩B'	A'∩B'	
			1

Examples:

These examples are taken from the Cambridge Essential Textbook which I am using to teach my students. It's an excellent resou rce

From a class of 30 students, 12 enjoy cricket (C), 14 enjoy netball (N) and 6 enjoy both cricket and netball.

Intersection

a Illustrate this information in a Venn diagram.

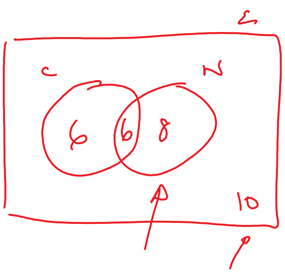
b State the number of students who enjoy:

- **i** netball only
- **ii** neither cricket nor netball

c Find the probability that a student chosen randomly from the class will enjoy:

- **i** netball
- **ii** netball only
- **iii** both cricket and netball

$$\begin{array}{r} 6 \\ 6 \\ 8 \\ \hline 20 \end{array}$$



b) (i) 8

(ii) 10

c) $Pr(N) = \frac{14}{30} = \frac{7}{15}$

$Pr(N \text{ only}) = \frac{8}{30} = \frac{4}{15}$

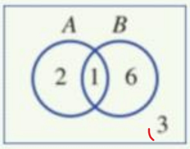
30 15

$Pr(\text{both}) = \frac{20}{30} = \frac{2}{3}$

Example 2:

These examples are taken from the Cambridge Essential Textbook which I am using to teach my students. It's an excellent resource.

The Venn diagram shows the distribution of elements in two sets, A and B.



$\cap$	A	A'	
B	1	6	7
B'	$\boxed{2}$	3	5
	3	9	$\boxed{12}$

a Transfer the information in the Venn diagram to a two-way table.

b Find:

- **i** $n(A \cap B) = 1$ ✓
- **ii** $n(A' \cap B) = 6$
- **iii** $n(A \cap B') = 2$
- **iv** $n(A' \cap B') = 3$
- **v** $n(A) = 3$
- **vi** $n(B') = 5$
- **vii** $n(A \cup B) = 9$

c Find:

- **i** $\Pr(A \cap B)$
- **ii** $\Pr(A')$
- **iii** $\Pr(A \cap B')$

$Pr(A \cap B) = \frac{1}{12}$

$Pr(A') = \frac{9}{12} = \frac{3}{4}$

$Pr(A \cap B') = \frac{2}{12} = \frac{1}{6}$