The addition rule

Saturday, 3 November 2018 3:17 pm

By the end of the lesson I would hope that you have an understanding of the following. I would also hope that you can apply the understanding to a number of different questions and question types

- Know what a Venn Diagram is
- Know how to read a Venn Diagram
- Know what a Karnaugh map is
- Know how to read a Karnaugh map
- Know what the addition rule is
- Know how to use the addition rule

RECAP:

In a previous lesson we looked at the idea of using Venn Diagrams to help us both describe/model events and to find probabilities.

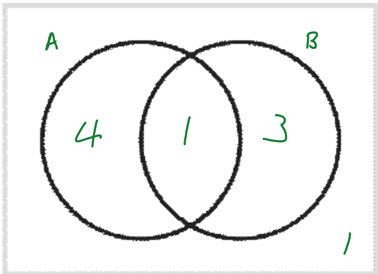
Venn Diagrams are awesome ...

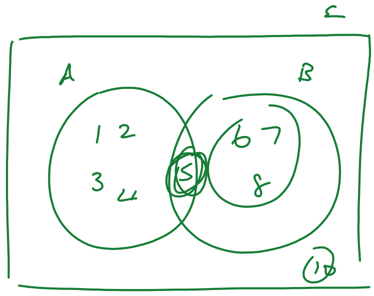
Here is an example:

$\cap$

$n(A \cap B)$

$P_r(A \cap B)$





$\varepsilon = \{1,2,3,4,5,6,7,8,10\}$

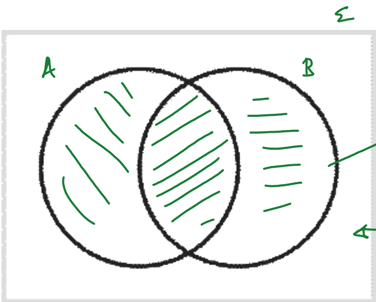
$A = \{1,2,3,4,5\}$

$B = \{5,6,7,8\}$

We can see, from the Venn Diagram that the data is **not** mutually exclusive.
We can tell this because there is an overlap between the circles.

Remember: Each section of the Venn Diagram stands for something!

$n(A \cap B')$



$n(A' \cap B)$

$n(A)$

$n(A')$

$n(A' \cap B')$

$n(A \cap B)$

This information can also be represented on a Karnaugh Map (or a **two way table**)

$\cap$	A	A'	
B	1	4	5
B'	3	2	5
	4	6	10

$n(B)$

$n(B')$

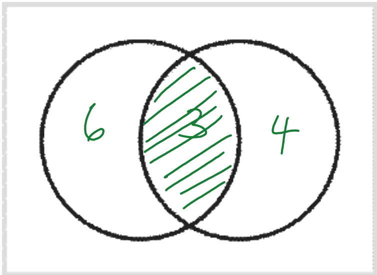
$n(A \cap B)$

$n(A \cap B')$

$n(A)$ $n(A')$

$n(A)$ $n(A')$

Using the Venn Diagram we can easily find the answer to $\Pr(A \cap B)$ or $n(A \cap B)$



$n(A \cap B) = 3$

$\Pr(A \cap B) = \frac{3}{13}$

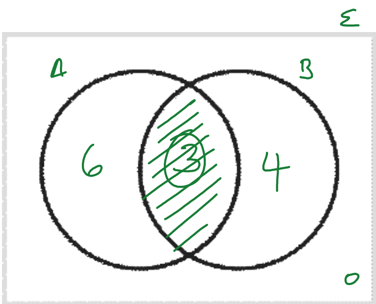
We can describe $\Pr(A \cup B)$ as the Probability of A or B happening.

Remember back to Year 7 when we used to tell you that when you work out the Probability of A OR B happening you found the probability of A happening, then the probability of B happening and then adding them together.

Not any more!

We need to be careful to ensure there isn't any overlap in the circles.

Look once again at the Venn Diagram:



$n(A \cup B) = 6 + 3 + 4 = 13$

$\Pr(A) = \frac{9}{13}$

$\Pr(B) = \frac{7}{13}$

$\Pr(A \cup B) = \frac{9}{13} + \frac{7}{13} = \frac{16}{13}$

Knowing the above, we can now write a formula called **The Addition Rule** to help us find out probabilities.

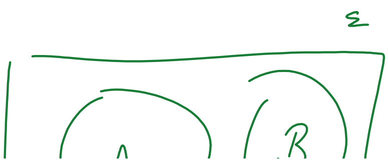
$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$$

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

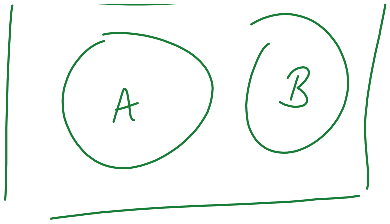
$$= 9 + 7 - 3$$

$$= 13$$

If the two events are **mutually exclusive** we can write the formula as:



$\Pr(A \cup B) = \Pr(A) + \Pr(B) \cancel{- \Pr(A \cap B)}$



$Pr(A \cup B) = Pr(A) + Pr(B) \cancel{- Pr(A \cap B)}$

$Pr(A \cup B) = Pr(A) + Pr(B)$

Example Questions

Question 1:

If we are told that $\Pr(A) = 0.6, \Pr(B) = 0.7, \Pr(A \cap B) = 0.5$ find $\Pr(a \cup b)$

$$Pr(A \cup B) = Pr(A) + Pr(B) - Pr(A \cap B)$$
$$= 0.6 + 0.7 - 0.5$$
$$= 1.3 - 0.5$$
$$= 0.8$$

Question 2:

A card is selected from a standard deck of 52 playing cards. Let A be the event 'the card is a diamond' and B be the event the card is a King.

52

4 suits

Diamond, Hearts || red

Clubs, Spades || black

13

2 - 10

Ace

King

Queen

Jack

A = Diamond

B = King.

Find:

A. $n(A)$ $= 13$

B. $n(B)$ $= 4$

C. $n(A \cap B)$ $= 1$

D. $\Pr(A)$ $= \frac{13}{52} = \frac{1}{4}$

E. $\Pr(A')$ $= 1 - Pr(A)$

$= 1 - \frac{1}{4}$

$= 3/4$

$= 3/4$

F. $\Pr(A \cap B)$ $= \frac{1}{52}$

G. Use the addition rule to find $\Pr(A \cup B)$

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$$
$$= \frac{13}{52} + \frac{4}{52} - \frac{1}{52} = \frac{\cancel{16}\,\cancel{8}\,4}{\cancel{52}\,\cancel{26}\,13} = \frac{4}{13}$$

H. Find the probability that the card is a king and not a diamond

4 $= \frac{3}{52}$